Consonant and Dissonant Waves

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**Description of Activity**

In this activity students will calculate the frequency of notes on the guitar, and determine if different combinations of notes are consonant or dissonant. The purpose of this activity is for students to demonstrate how a guitar fretboard functions and to understand the interaction between waves of different frequencies.

This lesson is appropriate for High School Physics or Guitar Building.

**Background**

Why do some notes sound good together? Consonant notes are notes that sound good when they are played together. Dissonant notes are notes that don’t sound good when they are played together. What makes a note consonant or dissonant? What are the properties of notes that make them consonant or dissonant? In this activity students determine the frequency of notes on the guitar. Students will determine whether or not different combinations of note frequencies are consonant or dissonant based on the ratios of the combinations. Students learned previously in another lesson the equation for the frequency of a wave, where:

speed = frequency\*wavelength

frequency = speed/wavelength

The frequency (Hz), or number of vibrations per second, for notes on a guitar can be determined using this helpful equation:

Hertz = 6.875 x 2 ^ ( ( 3 + MIDI Pitch ) / 12 )

The ^ symbol means ‘to the power of’. The MIDI Pitch value is set according to the MIDI standard, which is basically an arbitrary system of numbers assigned to each note up and down the scale for many different instruments on electronic devices. The values range, either from 1-128, or 0-127. For the purposes of this lesson, use the 0 - 127 value system where middle C (or, C5) has a MIDI Pitch value of 60. The lowest note upon a MIDI controller is a C and this is assigned note number 0. The C# above it would have a note number of 1. The D note above that would have a note number of 2. So "Middle C" is note number 60. A MIDI note number of 69 is used for A440 tuning, that is the A note above middle C, and the open A string on a guitar. Students can add or subtract values for each note up and down the scale from the A string.

Here is the hertz for middle C:

Hertz = 6.875 x 2 ^ (( 3 + 60 ) / 12 ) = 6.875 x 2 ^ 5.25 = 261.6255

The next note up, C#, is:

Hertz = 6.875 x 2 ^ (( 3 + 61 ) / 12 ) = 277.1826

And the next note, D:

Hertz = 6.875 x 2 ^ (( 3 + 62 ) / 12 ) = 293.6648

Let’s check note 69, the A note:

Hertz = 6.875 x 2 ^ ((3 + 69)/12) = 440

Students can use the value for frequency in hertz that they discover in this activity, and plug it into a graphing calculator to compare the frequencies of different notes on the guitar by using this equation:

y = sin (Hz (2πx))

**Learning Objectives:**

**(List measureable objectives)**

1. Students will identify the harmonic frequencies of the notes on a guitar by using the MIDI Pitch value of the notes and applying the given equation for determining the number of vibrations per second.
2. Students will describe whether a combination of notes (chords) are consonant or dissonant.
3. Students will model waveforms of consonant and dissonant notes.
4. Students will create a tablature chart of chords, and other combinations of consonant and dissonant notes.
5. Student will demonstrate chords, and combinations of consonant and dissonant notes on a guitar.

**Standards:**

List The Common Core Math, Next Generation Science Standard and/or SME Competency Gaps. For example:

[**CCSS.Math.Content.HSF-IF.C.7e**](http://www.corestandards.org/Math/Content/HSF/IF/C/7/e) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

[**CCSS.Math.Content.HSF-LE.A.2**](http://www.corestandards.org/Math/Content/HSF/LE/A/2) Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

**HS-PS4-1 Waves and their Applications in Technologies for Information Transfer.** Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

**Materials Required:**

* *“Notes on a Guitar Fingerboard” handout.*
* Blank fretboard handout.
* Graphing Calculator and/or Desmos.com (or similar website).

**Safety:**

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* N/A

**References:**

<http://www.physicsclassroom.com/class/sound>

<http://www.physicsclassroom.com/class/sound/Lesson-5/Guitar-Strings>

<http://musicmasterworks.com/WhereMathMeetsMusic.html>

<http://www.electronics.dit.ie/staff/tscarff/Music_technology/midi/midi_note_numbers_for_octaves.htm>

<http://www.philnewtonguitarlessons.co.uk/home/free-worksheets-1/1a>

**Activity:**

Warm-up activity:

Ask the students, “What is the formula for determining the frequency of a wave?” (Students learned about wave frequency in a prior lesson.)

*Answer: speed = frequency x wavelength, so, frequency = speed/wavelength.*

Show the video that demonstrate how the strings on a guitar actually create waves:

<https://www.youtube.com/watch?v=5X11Z4-fmE8>

(Any similar video will work; there are a lot online.)

*Alternatively, the waves on a guitar string can be demonstrated in class by using a strobe light and plucking the strings on the guitar while the strobe light is on.*

Review equation on notes and frequencies from previous lesson:

Equation to determine the frequency of notes:

fn = rn \* f0

f = note frequency

f0 = known frequency (example, A4 = 440 Hz)

n = number of steps up or down the scale from f0

r = 1.0595

**Say to the class:**

*You have observed how plucking the strings on a guitar will cause the strings to oscillate back and forth in waves. The mechanical vibration of the strings causes energy to travel through the air in the form of sound waves. These sound waves hit your ear at a certain frequencies causing you to hear the different sounds as each note is played.*

Next:

Provide an introduction to MIDI values, and introduce the equation to determine Hertz using MIDI values:

Hertz = 6.875 x 2 ^ ( ( 3 + MIDI Pitch ) / 12 )

Hand out “Notes on the Guitar Finger Board” and “Blank Fretboard.”

Identify with the class the open E (first string) and the C (second string, first fret). These two notes, when played together, constitute the most basic of C “chords.”

Ask the class: what makes a chord?

*Answer: Three or more notes that combine harmoniously?* (Technically C-E is not yet a “chord.”)

Ask: What makes notes harmonious?

*Answer: Consonance and dissonance. Intervals between notes are traditionally considered either consonant or dissonant. Consonant intervals are usually described as pleasant and agreeable. Dissonant intervals are those that cause tension and desire to be resolved to consonant intervals; in other words, they don’t sound good.*

Understanding how frequency and how waves work, will help to understand what makes notes consonant or dissonant. Note frequencies that match up at regular intervals “sound good” together, and are therefore consonant. Frequencies that don’t match up, don’t “sound good,” and are therefore dissonant.

Now, have the class determine the frequency of the C note, and E note. First determine the value using fn = rn \* f0

Then have the class check their work by using the MIDI value formula. *Note: MIDI value for C = 84, and MIDI value for E =88.You will also need to round to the nearest whole number.*

*C = 1047 Hz*

*E = 1319 Hz*

As a class, on the overhead, plug the values of C and E into Desmos.com using:

y = sin (Hz (2πx))

Explain that the ratio of E to C is about 5/4ths.  This means that every 5th wave of the E matches up with every 4th wave of the C. Analyze the wave diagram on the overhead, and notice how every cycle of 5 E waves matches up with every 4th C wave!

Have the students create some chords on the “Blank Neck Diagram.” Have them mark the note on the fret board, including the name of the note, the MIDI value of each note, and the frequency in Hertz. They should also identify whether or not the notes are consonant or dissonant. If there is a guitar in the class, have the students play their note combinations to see if they sound pleasant, or unpleasant.

**Quiz:**

1. What are consonant notes?

A. Notes that sound good when they are played together because they align at regular intervals.

B. Notes that don’t sound good together because they don’t align at regular intervals.

C. A single note that is played by itself.

D. Notes that sound good because they don’t align at regular intervals.

2. What are dissonant notes?

A. Notes that sound good when they are played together because they align at regular intervals.

B. Notes that don’t sound good together because they don’t align at regular intervals.

C. A single note that is played by itself.

D. Notes that sound good because they don’t align at regular intervals.

3. What is the formula used for calculating frequency using MIDI values?

A. 6.875 x 2 ^ ((3+ MIDI Pitch ) / 12 )

B. 440 x 2 ^ ((3+MIDI Pitch)/23)

C. 6.875 x 2 ^ ((3+440)/12)

D. 6.875 x 2 ^ ((3+MIDI Strike)/12)

4. Since the frequency for the open A string on the guitar is 440 Hz, calculate the frequency for the B note on the second fret of the A string using the formula fn = rn \* f0 (where r = 1.0595, n = the number of steps up or down the scale, and f0 = 440):

A. 80

B. 900

C. 932.36

D. 440

5. Since the frequency for the open A string on the guitar is 440 Hz, calculate the frequency for the C note on the third fret of the A string using the formula fn = rn \* f0 (where r = 1.0595, n = the number of steps up or down the scale, and f0 = 440):

A. 550

B. 1398.54

C. 932.36

D. 777

6. The formula for determining the speed of a wave is:

A. frequency = speed/wavelength

B. speed = base\*width\*height

C. speed = 55 mph

D. speed = frequency\*wavelength

7. The formula for determining the frequency of a wave is:

A. speed = frequency\*wavelength

B. the speed of light

C. frequency = speed/wavelength

D. unknown

8. True or False, the E note and C notes are dissonant?

A. True B. False

9. Using the formula 6.875 x 2 ^ ((3+ MIDI Pitch ) / 12 ), calculate the frequency of a note with a MIDI Pitch value of 69. Show your work:

A. 550

B. 660

C. 110

D. 440

10. What note has a value of 440 Hz?

A. B5

B. C4

C. A4

D. D5

**Quiz Answers**

1. A

2. B

3. A

4. C

5. B

6. D

7. C

8. B

9. D

10. C

**Reviewing Faculty Cohort Members:**

* n/a