Microcantilevers Model Activity: Resonant Frequency vs. Mass
Instructor Guide

Notes to Instructor

This Activity is part of the Microcantilevers Learning Module. It is participants read about the various applications of microcantilevers first. Also, "How Does a Cantilever Work?" should be required reading as it explains much the theory explode in this activity.

There is an activity kit available from SCME. The kit can be ordered via the SCME website. (http://scme-nm.org)

The Microcantilevers Learning Module consists of the following:

- *Book 1*
- Learning Module Map for Instructors
- Microcantilever Knowledge Probe
- Microcantilever Applications Overview (PK)
- Chemical Sensor Arrays (PK)
- Atomic Force Microscopes (PK)
- *Book 2*
- How Does a Cantilever Work? (PK)
- **Microcantilever Model Activity: Resonant Frequency vs. Mass (SCME Kit Available)**
- Microcantilevers Terminology and Research Activity
- Final Assessment

Description and Estimated Time to Complete

This activity provides a procedure that will allow you to further explore the motion of a cantilever under a varying mass and to determine the relationship that expresses the resonance frequency of a cantilever as a function of mass. This activity simulates the dynamic mode of operation for microcantilevers used in MEMS (microelectromechanical systems) sensors.

Estimated Time to Complete
Allow approximately 1.5 hours to complete this activity in class. The report will take an additional 2-6 hours depending on your experience and existing knowledge of cantilever operation.

Introduction

In many MEMS applications, a change in the resonant frequency of a cantilever structure indicates a change in the cantilever system mass. The system includes the cantilever structure itself plus any mass added to the structure. For example, in a chemical sensor array (CSA) that contains an array of microcantilevers, target molecules bind with the surface probe molecules of the oscillating cantilevers. This causes a change in the mass of each cantilever system and in turn, their natural frequency of oscillation. These frequencies are monitored and any changes are converted to electrical outputs which represent the amount of accumulated mass on the cantilevers' surfaces.

In this activity, you will simulate a single cantilever of a CSA by adding mass and observing the change in the system's natural frequency.

Activity Objectives and Outcomes

Activity Objectives

- Determine the relationship between the resonant frequency of a cantilever system to added mass.
- Explain the similarities between the physical characteristics of the macrocantilever model and the microcantilever-based chemical sensor array.

Activity Outcomes

Through experimentation, data collection and analysis, you will explore the relationship between mass and its effect on the resonant frequency of a cantilever. At the end of this activity you should be able to answer the following questions:

- How is the natural resonant frequency of a mechanical system affected by the addition of mass?
- How do you determine the frequency of a system from digital video data?
- How do you determine if the Frequency vs. Mass Added relationship is linear or non-linear?

Attitude & Behavior

In this activity, consistency, repeatability, and attention to detail are needed to ensure the most accurate and repeatable outcome.

Team

This activity should be performed in teams of 2 to 3 participants to ensure the best outcome. The multiple tasks for some of the procedure steps require more than one set of eyes and one set of hands; therefore each team should assign a specific task to each individual on the team. These tasks can be rotated at various points in the activity so that all participants experience each part of the experiment.

Materials

Supplies provided in the SCME kit

- Several sticks of different lengths, widths, thicknesses, and/or materials.
- Clamps (Large enough to clamp the stick to a table)
- Binder clips (unit masses – do not need to weigh as they are all the same and considered *unit masses,* analogous to individual molecules or particles adhering to a micro cantilever sensor).

Supplies provided by the instructor and/or participants

- Digital Camera with video capability – 30 frames/sec is best
- Camera tripod
- Computer with spreadsheet software, Apple QuickTime software, and the software needed to transfer the video files from the camera.
- A sturdy table to clamp the sticks (cantilevers) to.

Preparation/setup

Cantilever in front of white board with reference lines. This activity can be done with any type of individual planks or sticks. The equations apply to any cantilever structure that has a rectangular cross section of thickness, t, width, w and length, l.

This activity can be performed in any classroom with a sturdy flat table and a computer with the required software (see materials). To make recording the data easier, place a background within the field of view that can be used as a reference. Such a device could be a file cabinet, clock, set of window panes, or white board with a few horizontal lines (see picture above). Points on the reference device allow you to better identify the specific points of an oscillation within a video frame.

Before getting started, review the Documentation Supplement at the end of this activity.

Documentation

Write a formal lab report for this activity. Report should include the following components:

- Title
- Author(s)
- Objectives / Abstract
- Introduction (Background and Pre-Lab Questions)
- Materials / Equipment
- Setup Description (Sketch or pictures of the experimental setup)
- Pre-Activity questions / answers
- Procedure: Each procedure step with outcomes where applicable
- Summary of observations and results
- Video to support outcomes
- Data, graphs, tables to support outcomes
- Analysis / Discussion
- Conclusions
- Post-Activity questions / answers
- References (when applicable)

A Formal Lab Report Sample is provided at the end of this activity.

Pre-Activity Questions

1. What is frequency?

2. What is the natural resonant frequency of a system?

3. What do you expect to happen to the natural frequency of the cantilever system as you add mass to it? (Hint: Think of a child vs. an adult on a diving board)

4. Will the natural frequency of the cantilever oscillation increase or decrease with added mass?

5. With your digital camera set for video, how many frames per second does it run?

Pre-Activity Questions / Answers

1. What is frequency?
   Answer: cycles or oscillations per second, the number of times the cantilever oscillates in one second. The unit of frequency is Hertz (Hz) which can be written as $\boldsymbol{Hz = 1/sec}$

2. What is the natural resonant frequency of a system?
   Answer: Any object which is free to oscillate will do so at its natural frequency. If you push or strike an object, it will vibrate at its natural frequency. For example, a tuning fork will resonate at a specific frequency when struck.

3. What do you expect to happen to the natural frequency of the cantilever as you add mass to it?
   Answer: The resonant frequency will decrease when mass is added to the cantilever.

4. Will the natural frequency of the cantilever oscillation increase or decrease with added mass?
   Answer: Decrease

5. With your digital camera set for video, how many frames per second does it run?
   Answer: Each camera will be different but can be determined in the camera's specifications or through experimentation. If you acquire a video of a clock with a second hand, you can subsequently count the number of video frames it takes to go from one second to the next. Generally, the cameras run at either 15 frames a second or 30 frames a second. If you count the number of frames over several seconds, your result will be more accurate. For example, you may only count 29 frames over one second but 149frames over a period of 5 seconds.

Procedure: Microcantilever Model Activity- Resonant Frequency vs. Mass

The activity allows you to discover a functional relationship through experimentation, data collection and analysis.

Description

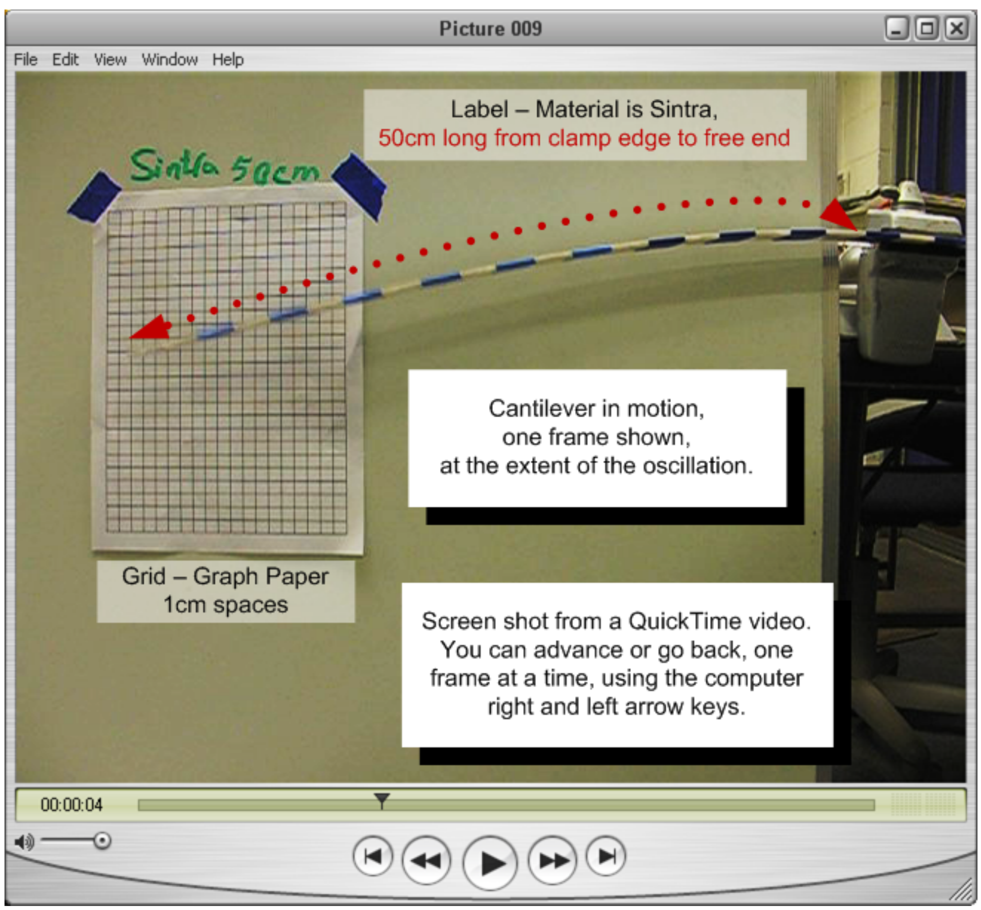
This procedure will use cantilever sticks, clamp(s), binder clips, and a video camera to determine the effect that mass has on the resonant frequency of a cantilever.

Use the Document Supplement and Lab Report sample provided at the end of this activity to help you gather and analyze the outcomes of each step of this procedure.

1. Set up cantilever

Description Place a sturdy table in front of a background within the field of view that can be used as a reference, something with a grid or lines. You can print out graph paper of any specific grid spacing at Incompetech or create your own.

Using the clamp, secure one end of the cantilever stick to the table to make your cantilever system. The length of the meter stick should extend in front of and across your reference device. See figure below.



2. Set up camera

Description Secure the camera in a position that will allow it to record the movement of the cantilever with the reference device as background. A tripod is extremely useful for this task.

3. Record cantilever specifications

Description Record the cantilever's specifications:

- Length between the clamp (fixed) and free end.
- Thickness
- Width
- Type of material

4. System Calibration

Description

- Place the camera on the tripod.
- Put a clock with a second hand in the field of view. (A digital clock that shows seconds could also be used).
- Record (video) the clock for 5 seconds.
- Download the video from the camera to the computer.
- View the video in a viewer (QuickTime, free viewer download) that allows you to step through the video frame by frame.
- Count the number of frames over a five-second interval, second hand iterations. Start counting when the second hand just starts to move (or in the case of a digital camera, as the second number display just starts to change to the next second).

5. Create a data table

Description Create a table using a spreadsheet such as Microsoft Excel in which to record your data. The table should be set up to record frequency as a function of mass added. See example below for a camera having 30 frames per second.

Camera Frames/Second = 30 frames/second, length = 50cm, thickness = 3mm, width = 5cm, material: Sintra			
# clips added	# Oscillations	# Frames	Frequency = Osc/Sec = Hz
Example	5	38	6.33
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Table 1: Frequency as a Function of Mass

6. Record cantilever oscillations with no mass

Description
- Ready the camera to record the first oscillation.
- Start recording.
- Set the cantilever in motion by pushing down (apply force) on the open end of the cantilever, then releasing. This is similar to plucking a guitar string.
- Record for several oscillations – at least 5 complete oscillations should be recorded.

NOTE: This step gives you the data needed to determine the natural resonant frequency for the cantilever before mass is added.

7. Record cantilever oscillations with added mass

Description
- Add a binder clip to the free (suspended) end of the cantilever.
- Start video record.
- Set the cantilever in motion as before.
- Stop the video
- Transfer the video to the computer

Note: Add the masses at the end of the cantilever only, do not spread out the masses over the entire length of the cantilever.

8. Determine the natural resonant frequency of the cantilever system

Description
- View the video segments.
- Note the number of complete oscillations, number of frames associated with this number.
- Calculate the frequency in Hz.

$$f = \frac{(\#oscillations) * (frames/second)}{(\#frames)}$$

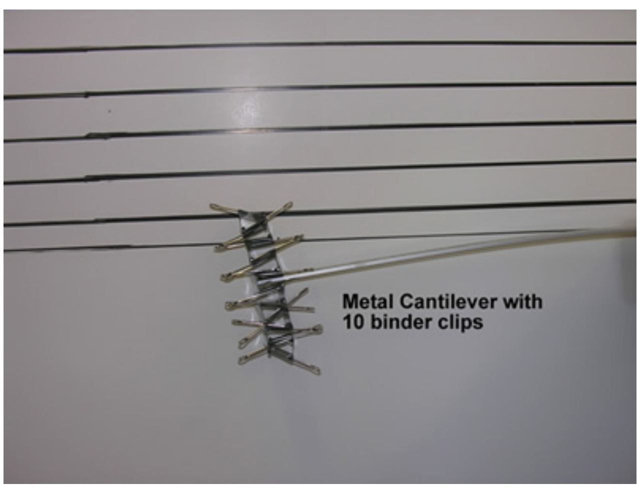
NOTE: If you were unable to get consistent values, evaluate your setup and procedure. Make any necessary adjustments and repeat steps 6 and 7.

9. Record your data

Description Record the raw data for each mass added. (Number of frames per oscillations – you can count the number of frames for several oscillations to get a more accurate value).

10. Repeat steps 7 – 9

Description Repeat steps 7 – 9 for up to 10 binder clips or as many as you can put on the meter stick. Attach all the clips as close as you can to the first one. *(See picture below)*



11. Plot your data

Description When all the data has been collected, plot a line graph that shows the frequency vs. mass added to the cantilever. If you are using Excel, the Chart Wizard makes this step easy. See the documentation supplement at the end of this unit for an example.

12. Complete the documentation requirements

Description Write a formal lab report for this activity. Report should include the following:

- Title
- Author(s)
- Objectives / Abstract
- Introduction (Background and Pre-Lab Questions)
- Materials / Equipment
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- Analysis / Discussion
- Conclusions
- Post-Activity questions / answers
- References (when applicable)

13. Answer all of the Post-Activity Questions and turn in with the report.

Activity Variations – May be required by the instructor in addition to the Frequency Vs Mass added component of this lab.

To further explore the effect that mass has on resonant frequency, you can change the specifications of the cantilever. Repeat this activity for one or more of the following. However, change only ONE specification for each activity.

<u>Use a cantilever stick of a different material – metal, wood, plastic. The kit comes with two different materials.</u>

- Complete this activity with 2 or 3 different materials.
- Discuss the effect that "cantilever material" has on this activity's outcomes.

Note: the material property call Young's Modulus of Elasticity, E, determines the "springiness" of the material. The spring constant, k, is determined by both E and the shape of the cantilever. The equations listed here assumes a rectangular cross section of width, *w*, and thickness, *t* for a given length, *l*.

<u>Frequency vs. length</u>

- Complete this activity with 5 to 10 different lengths of cantilevers.
- Put the data in a table and graph accordingly.
- Discuss the effect that "cantilever length" has on the natural frequency of the system.
- How does this compare to the mathematical model? Can you plot the theory with the measured data on the same graph?
- Plot Frequency vs. Length for at least one cantilever.
- As time permits, acquire data for a cantilever of different thickness, how does the curve change?
- What about a cantilever of different widths? Materials?

Frequency vs. Thickness

- Glue two sticks together to double the thickness, or use a stick of double thickness and having the same width and length. You only want to change one variable at a time. Changing thickness and length or width at the same time will confound your results. Note: Doubling the thickness also doubles the mass! What change in frequency do you expect to see as you double the thickness?
- What about tripling the thickness? How does the frequency change?
- Create a table and graph showing your results.
- Discuss the effect that "cantilever thickness" has on the resonant natural frequency of the system. How does this compare to theory (equation)? Try to plot the predicted results with the actual data.

Post-Activity Questions

1. Does the graph of frequency vs. mass added represent a linear relationship (is it a straight or curved line)? What do you predict from the equation?

2. Is the frequency decreasing or increasing with added mass? Why?

3. Based on the graph of your actual data, how would you state the relationship between frequency and mass added? In other words, is it an inverse relationship? Inverse squared? Inverse square root? Squared? Direct proportion?

4. Your frequency vs. mass added graph (number of clips) can be used to determine the mass of an unknown sample. Say you add several coins to the end of the cantilever and determine the frequency of the system. How would you determine the mass of the coins in "clip units" from the graph?

5. Study the Microcantilever unit on Chemical Sensor Arrays (CSA) *(if you haven't already)*. Write a short discussion on how this activity simulates a dynamic mode CSA. Discuss how you could "tune" your sensor for a given situation. For example, if you wanted to make the cantilever work in a lower frequency range, how would you change its design?

   Ideas for discussion:
   a. How could you tell how much additional mass has been added to the cantilever based on the observed frequency shift? (Take a look at your curve. Imagine you are reading a shift in the output of a CSA, how would you determine the amount of material adhered to the cantilever?)
   b. How would you adjust the cantilever length and/or thickness to make the resonance match an off the shelf electronics package? This is critical in design since designing a new electronics package in addition to a cantilever device becomes more costly.

6. What effect does the cantilever's material have on its frequency? (Prove your answer using the E (Young's Modulus) for at least 3 different materials). Note: E is usually given in units of GPa, or Giga Pascal's – this is a pressure and stress unit, force/area. Definition of a Pascal:
7.

$$1Pa = \frac{1kg}{m * s^2} = 1\ N/m^2$$

For most materials, E is given in GPa or kN/mm^2

E = 9.6GPa for Sintra, a type of PVC Foam. This was determined assuming a $1.4 g/cm^3$ density, 3mm thickness, 50cm length, 5cm width and measuring a 2.5Hz resonant frequency for this cantilever at this length.

   a. What would happen to the frequency if the cantilever was made of Aluminum, having an E=69GPa and density $\rho = 2.7g/cm^3$?
   b. What about steel which has an E = 200 GPa and density $\rho = 7.9g/cm^3$?
   c. And one of the stiffest materials, diamond, E=1220 GPa and density $\rho = 3.5g/cm^3$?

8. Based on the following equation, what variables affect a cantilever's frequency?
    a. Does density?
    b. Length?
    c. Width?
    d. Thickness?
    e. Young's Modulus of Elasticity?

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}$$

where

$$k = \frac{Et^3 w}{4l^3}$$

Note: Omega (ω) is in radians per second and frequency is in cycles or oscillations per second. There are 2π radians in one cycle.

Post-Activity Questions / Answers

1. Does the graph of frequency versus mass added represent a linear relationship (is it a straight or curved line)? What do you predict from the equation?

   Answer: This will depend on each team's graph and scale, but it should not be linear. You can re-write the equation as:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{M + m_{added}}}$$

   The frequency should decrease with mass added, but with the inverse square root – meaning the curve won't be linear but a curved relationship.

2. Is the frequency decreasing or increasing with added mass? Why?

   Answer: Decreasing – it is in the denominator, therefore, adding mass results in the frequency decreasing.

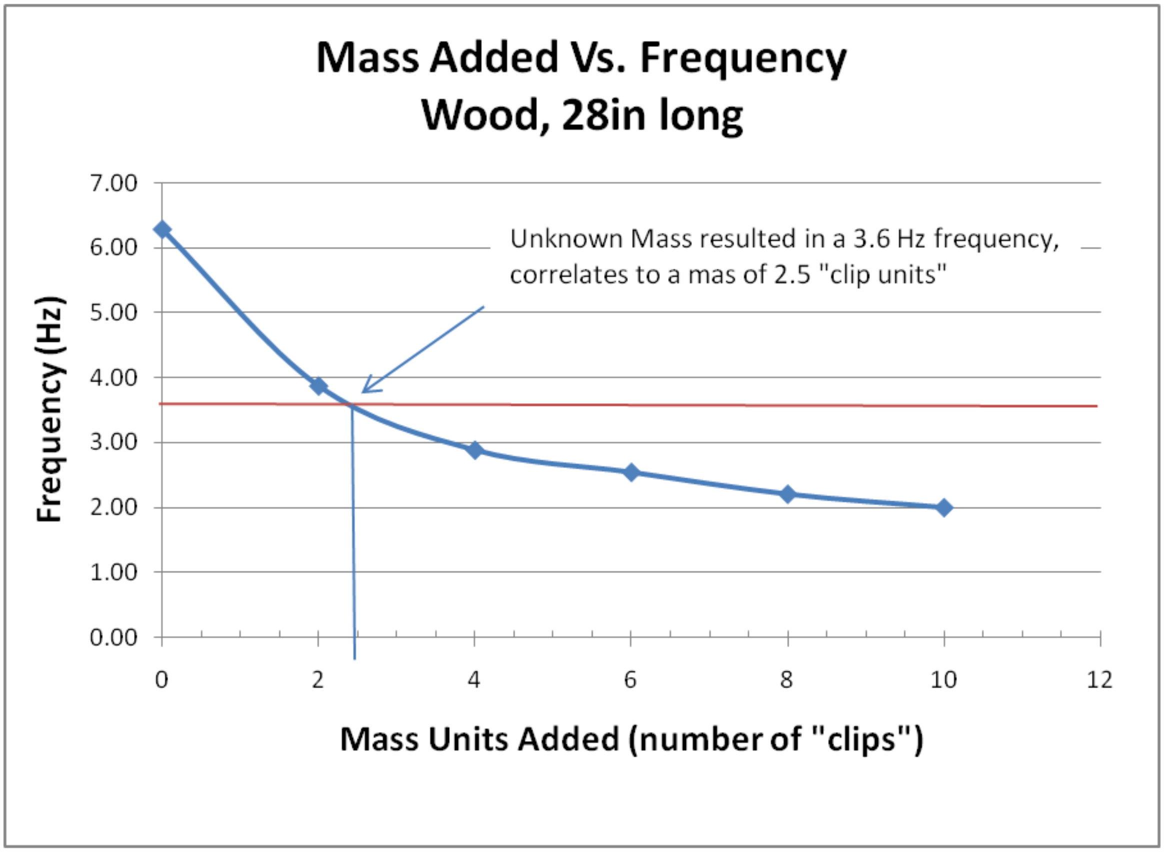
3. Based on the graph of your actual data, how would you state the relationship between frequency and mass added? In other words, is it an inverse relationship? Inverse squared? Inverse square root? Squared? Direct proportion?

   Answer: Resonant frequency has an inverse relationship to mass. As the mass increases, the resonant frequency decreases and vice versa. It is a curve and is not a straight line. The resonance frequency is related to the inverse square root of the mass added.

   1. $$\mathrm{f} \approx \frac{1}{\sqrt{\mathrm{m_{added}}}}$$

4. Your frequency Vs mass added graph (number of clips) can be used to determine the mass of an unknown sample. Say you add several coins to the end of the cantilever and determine the frequency of the system. How would you determine the mass of the coins in "clip units" from the graph? You may wish to make a sketch to help explain this.

   Answer: You can draw a horizontal line from the frequency measured of the unknown mass. Draw a vertical line from where the horizontal line intersects the curve. The vertical line will cross the x-axis and give you the mass of the unknown in "clip units."



5. Study the Microcantilever unit on Chemical Sensor Arrays (if you haven't already). Write a short discussion on how this activity simulates a dynamic mode CSA. Discuss how you could "tune" your sensor for a given situation. For example, if you wanted to make the cantilever work in a lower frequency range, how would you change its design?

   Answer: You can reduce the frequency by making the cantilever longer. If you double the length, for example, you will effectively reduce the operating frequency range by about ¼. Width doesn't change the resonance frequency at all for the base unit. However, if the width is doubled, the relationship between resonance frequency vs. mass added will show the system is not as sensitive! Think about it – the denominator includes the $+m_{added}$ *, hence, if the mass of the cantilever itself, M, dominates, is large, then adding "m" will only change the frequency by a relatively small amount. On the other hand, reducing the width of the cantilever reduces M and therefore, the mass added will play a more significant role.*

Ideas for discussion:

How could you tell how much additional mass has been added to the cantilever based on the observed frequency shift? (Take a look at your curve. Imagine you are reading a shift in the output of a CSA, how would you determine the amount of material adhered to the cantilever?)

This was addressed previously.

How would you adjust the cantilever length and/or thickness to make the resonance match an off the shelf electronics package? This is critical in design since designing a new electronics package in addition to a cantilever device becomes more costly.

Answer: The thickness and /or length could be adjusted to match the electronics frequency range of operation. Usually it is easier to adjust the length. Changing the thickness requires a deposition process change while a change in width or length requires a mask change.

What effect does the length of the cantilever have on its frequency?

Answer: Inverse relationship – as the length increases the frequency decreases and vice versa.

6. What effect does the cantilever's material have on its frequency? (Prove your answer using the E (Young's Modulus) for at least 3 different materials). Note: E is usually given in units of GPa, or Giga Pascal's – this is a pressure and stress unit, force/area. Definition of a Pascal:

$$1Pa = \frac{1kg}{m * s^2} = 1\ N/m^2$$

For most materials, E is given in GPa or kN/mm^2

E = 9.6GPa for Sintra, a type of PVC Foam. This was determined assuming a 1.4g/cm^3 density, 3mm thickness, 50cm length, 5cm width and measuring a 2.5Hz resonant frequency for this cantilever at this length.

a. What would happen to the frequency if the cantilever was made of Aluminum, having an E=69GPa and density $\rho = 2.7g/cm^3$?

Answer: If you keep all the geometric dimensions of the cantilever the same, l, w and t, then you can write the equation as:

$$f_1 = C\sqrt{E_1/\rho_1}$$

And

$$f_2 = C\sqrt{E_2/\rho_2}$$

Where C is the same constant for both materials. E and ρ are the material properties.

Take the ratio of these two equations:

$$\frac{f_2}{f_1} = \frac{\sqrt{E_2/\rho_2}}{\sqrt{E_1/\rho_1}}$$

Or

$$f_2 = f_1\sqrt{\rho_1/E_1} * \sqrt{E_2/\rho_2}$$

Since we know all the numbers on the right side of the equation, we can solve for f_2.

Aluminum cantilever would have a resonant frequency of about 4.83Hz

b. What about steel which has an E = 200 GPa and density $\rho = 7.9g/cm^3$?

Answer: expect about the same as aluminum, comes out to 4.80Hz. The E/ρ ratio is almost the same for steel as aluminum.

c. And one of the stiffest materials, diamond, E=1220 GPa and density $\rho = 3.5g/cm^3$?

Answer: about 18Hz, ρ is about 1.5 that of Sintra, but E is about 120 times higher. Doing the math, you find that the frequency is about 6 times higher.

7. Based on the following equation, what variables affect a cantilever's frequency?
   a. Does density?
   b. Length?
   c. Width?
   d. Thickness?
   e. Young's Modulus of Elasticity?

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}$$

where

$$k = \frac{Et^3w}{4l^3}$$

Note: Omega (ω) is in radians per second and frequency is in cycles or oscillations per second. There are 2π radians in one cycle.

Answer: All of the above with the exception of width affect the cantilevers natural frequency. If you realize that the mass is just density multiplied by volume:

$$m = \rho V = \rho * l * w * t$$

Then substituting this into the equation with k:

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{\left[\frac{Et^3w}{4l^3}\right]}{\rho lwt}} = \sqrt{\frac{E}{\rho} * \frac{t^2}{4l^4}} = \frac{t}{2l^2}\sqrt{\frac{E}{\rho}}$$

Width doesn't play a role, only thickness, length, Young's Modulus of Elasticity and the density of the material.

Summary

This activity simulates the output of a cantilever in a dynamic mode Chemical Sensor Array. The binder clips represented unit masses such as molecules, viruses, DNA snippets, proteins, and antibodies that attach to the surface of the cantilevers. As these molecules attach to each of the cantilevers' surfaces, the cantilevers' effective mass changes resulting in a change in resonant frequency. The change in resonant frequency indicates a change in the concentration of target material in the sample being evaluated or the amount of time the cantilevers were exposed to the sample environment.

Microcantilever Model Activity – Documentation Supplement

As part of the Microcantilever Model Activity: Resonant Frequency Vs. Mass, this supplement will assist you in

1) gathering the correct data for your laboratory report (documentation), and
2) clarifying what is to be included in your lab report (documentation template provided).

Before starting on this Lab, you should answer the Pre-Activity Questions.

To obtain accurate data, you will be acquiring data through the use of a video camera or a digital camera with video capabilities. You will capture video for each experiment, and then review it frame by frame to determine the resonant frequency of the dynamic cantilever system. It is highly recommended that you go through the steps of data acquisition and analysis at least once before acquiring all the video and then trying to analyze it.

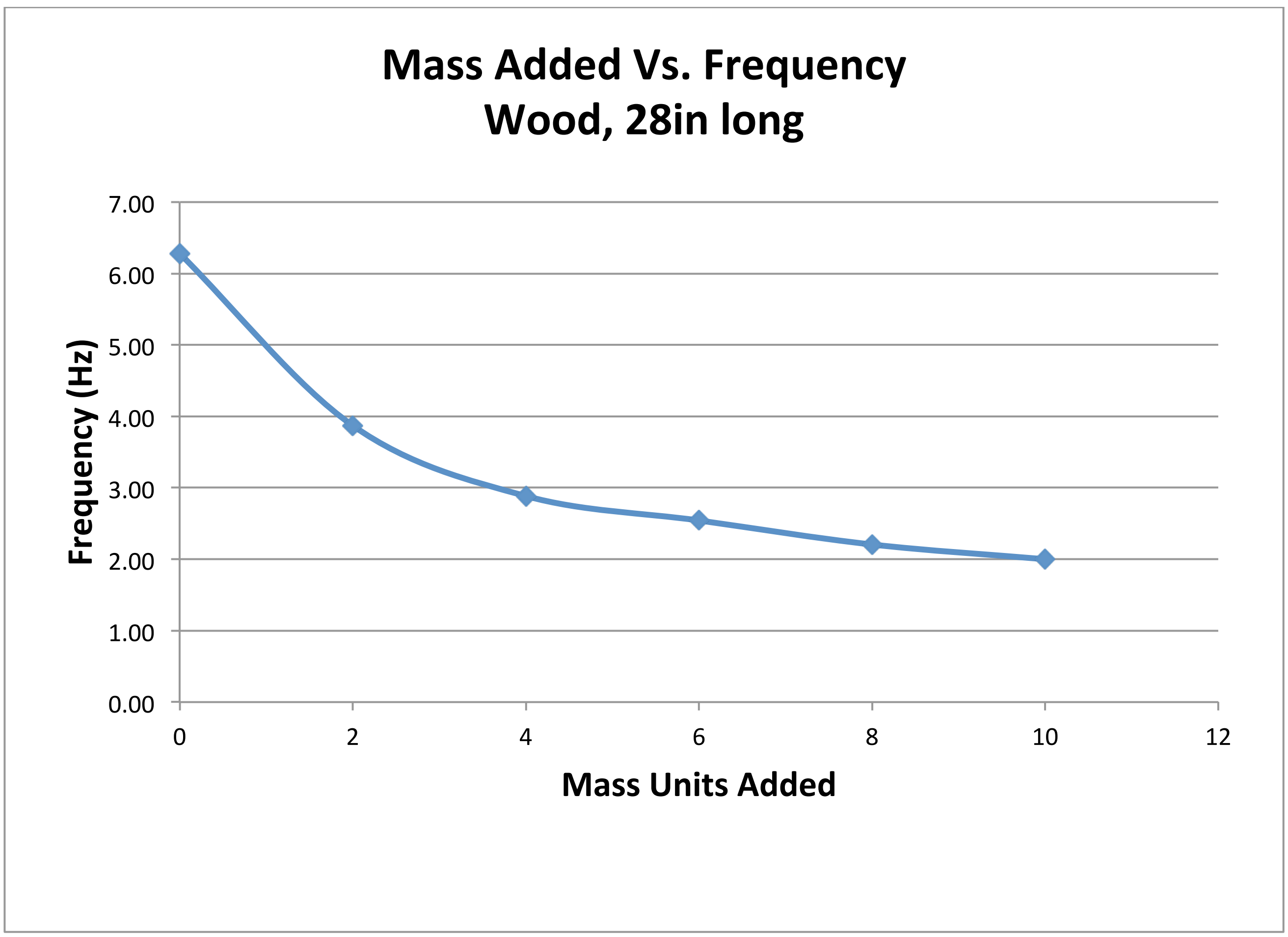
Follow the steps in the Procedure section of this activity.

To determine frequency, you need to combine the number of oscillations observed, the number of frames counted for those number of oscillations and knowledge of the frame rate of your camera. The frame rate can be easily found by capturing several seconds of a watch and counting the number of frames for a given number of seconds, and then calculating frames/second.

Below are examples of chart and data tables that illustrate the outcomes of a similar experiment.

Experimental Run Number:	2
Material:	Wood
Length:	28in
Thickness:	t
Width:	w
Camera Frames / Sec:	30

Wood Video File	# Clips Added	Number of Oscillations	Number of Frames	Wood Frequency (Hz)
1588	0	9	43	6.28
1598	2	8	62	3.87
1599	4	5	52	2.88
1600	6	5	59	2.54
1601	8	5	68	2.21
1602	10	5	75	2.00



Title

Authors

Grading Criteria: All sections in order, well-formatted, very readable. All grammar/spelling correct and very well-written

Objectives or Abstract

Grading Criteria: Abstract contains reference to all major aspects of carrying out the experiment and the results, well-written. Should not be more than 250 words.

Introduction (Background and Pre-Lab Questions)

Here you can include some of the background related to this experiment including theory and equations.
Frequency is related to the spring constant of the system, k, and the system's mass:

$$\omega = 2\pi f = \sqrt{\frac{k}{M}}$$

Equation 1

The spring constant is determined by the material property, E, call the *Bulk* or *Young's Modulus*, combined with information on how the material is distributed. In this case, the material is uniformly distributed in a rectangular cross section having thickness, t, width, w, and length, l.

$$k = \frac{E\,t^3 w}{4l^3}$$

Equation 2

Grading Criteria: Introduction complete and well-written; provides all necessary background principles for the experiment

Materials

Setup Description

Photographs of the setup can be used. Use a photo editor crop and to add labels if needed. It is good practice to include captions.

Procedure

Step by step instructions on how the experiment was done.
Grading Criteria: Well-written in paragraph format, all experimental details are covered

Observations

Observations made during the experiment.

Data Tables, Graphs

Collection of tables and graphs. Label all tables and make sure the reader knows what each represents. Graph axes must be labeled clearly, units included and graphs containing multiple data sets must include clear legends. Graphs should have captions summarizing what is being shown. Graphs and Tables need to have Figure and Table numbers so that the author(s) can reference the graphs in the text.

Grading Criteria: All figures, graphs, tables are correctly drawn, are numbered and contain titles/captions.

Analysis/Discussion

This is where the author(s) summarize the graphs and tables in text. "From figures xx and yy we see that as the cantilever has mass added to it, the resonance frequency of the system decreases in a non-linear manner as is expected from Equation 1 above."

Grading Criteria: All important trends and data comparisons have been interpreted correctly and discussed, good understanding of results is conveyed

Conclusion

Grading Criteria: All important conclusions have been clearly made, student shows good understanding

References (Optional)

Appendices (Optional)

Support for this work was provided by the National Science Foundation's Advanced Technological Education (ATE) Program.