NACK NETWORK

Building College-University Partnerships for Nanotechnology Workforce Development

Solar Cell Optics

Outline

- What is Light?
- Light Absorption
- Interaction of Light with Matter
  - Reflection/Refraction
  - Fabry Perot resonances
  - Waveguiding
  - Diffraction / Photonic Crystals
  - Plasmonics
  - Mie Scattering

What is Light?

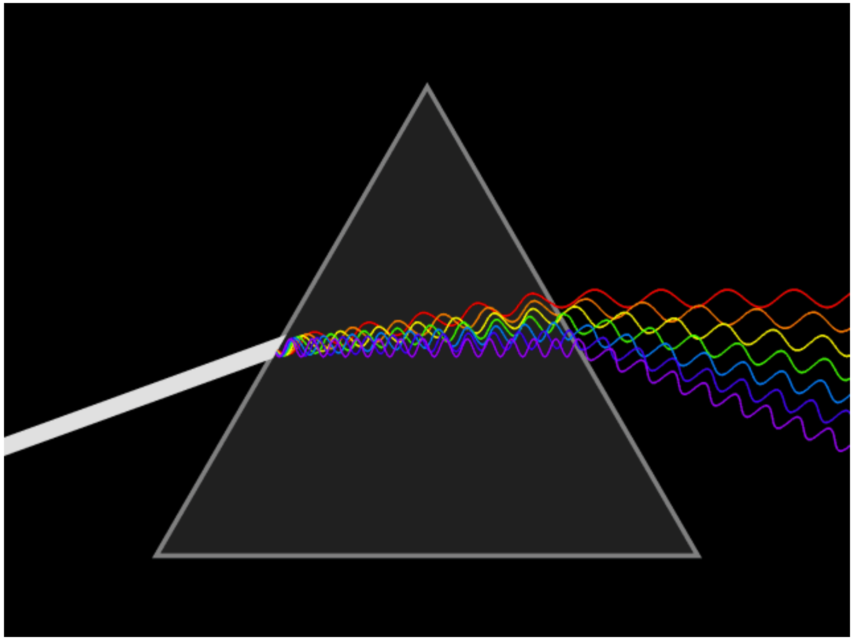


Fig. 1

Light: Electromagnetic wave radiation

- Fig. 1 taken from Wikipedia: http://en.wikipedia.org/wiki/Wavelength#mediaviewer/File:Light_dispersion_conceptual_waves.gif

What is Light?

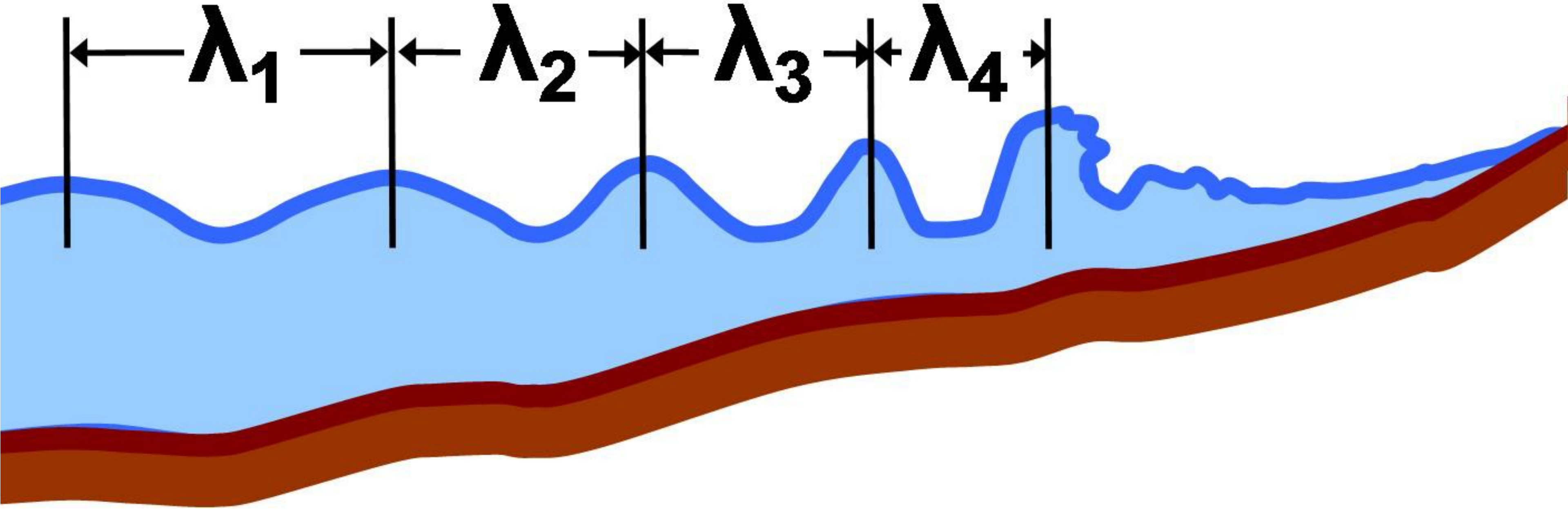


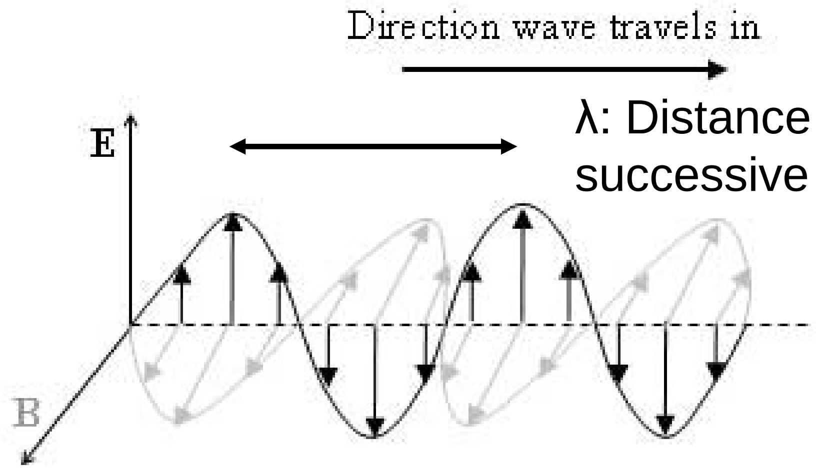
Fig. 2

Resemblance to ocean waves?

- Fig. 2 taken from Wikimedia: http://upload.wikimedia.org/wikipedia/commons/d/d7/Local_wavelength.JPG

What is Light?

E: Electric field
B: Magnetic flux density
c: speed of light
f, v: frequency of light
h: Planck's constant
ω: angular frequency



λ: Distance between successive peaks

f: number of cycles per second passing a fixed point

Fig. 3

$$\lambda = \frac{c}{f} = \frac{c}{\nu}$$: wavelength

$$E = h\nu = \frac{h}{2\pi} 2\pi f = \hbar\omega$$: Energy

- Fig. 3 original: http://www.sipex.aq/access/page/index.html%3Fpage=03040cd0-bfaa-102a-8ea7-0019b9ea7c60.html

What is Light?

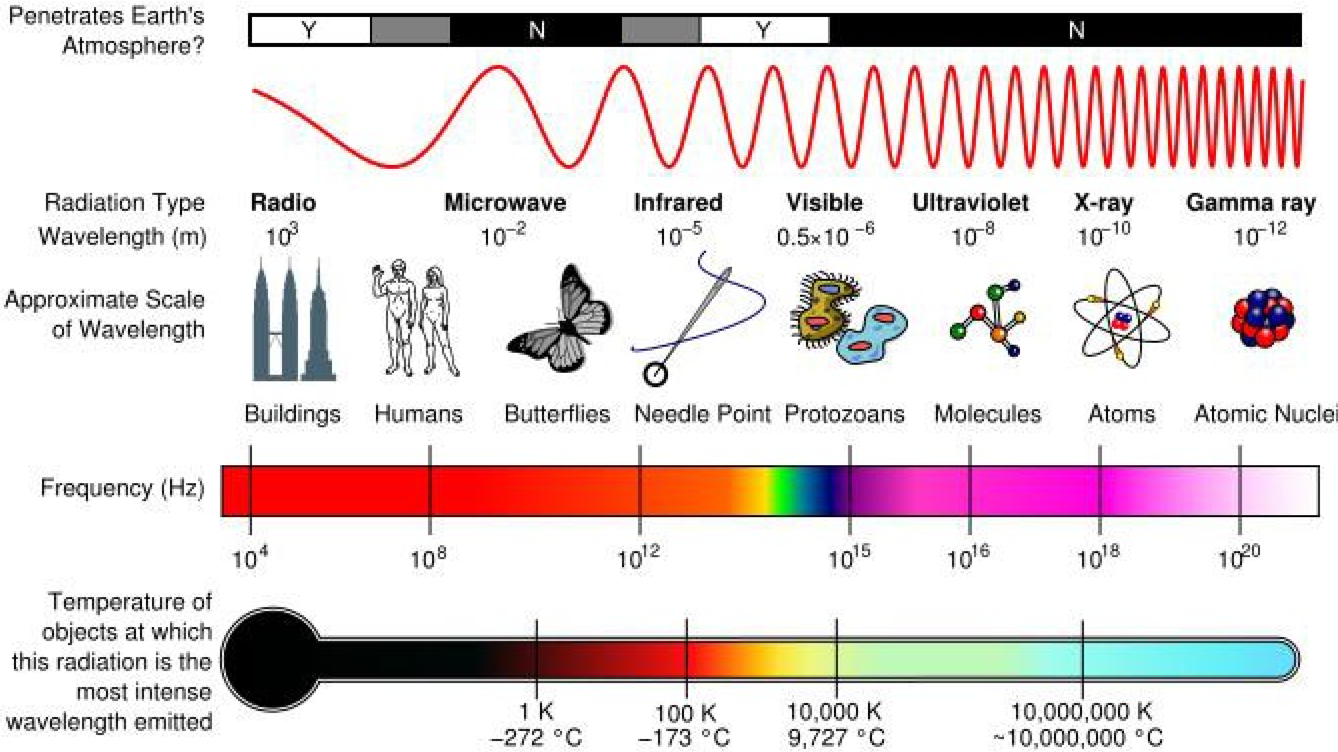
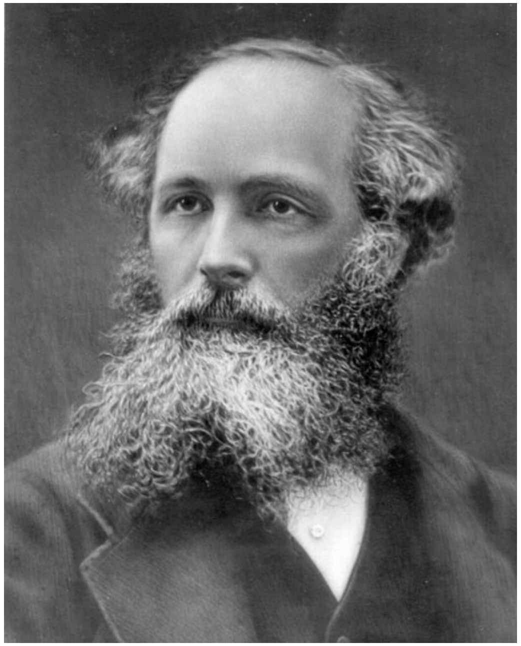


Fig. 4

- Fig. 4 taken from: https://wikis.engrade.com/a121biology2012/visualcommunication

What is Light?



James Clerk Maxwell

(original image: http://www.st-andrews.ac.uk/~ulf/perfectimaging.html)

$$\nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t} - \vec{M}(\vec{r},t)$$

$$\nabla \times \vec{H}(\vec{r},t) = \frac{\partial \vec{D}(\vec{r},t)}{\partial t} + \vec{J}(\vec{r},t)$$

$$\nabla \cdot \vec{B}(\vec{r},t) = \rho_m(\vec{r},t)$$

$$\nabla \cdot \vec{D}(\vec{r},t) = \rho_e(\vec{r},t)$$

Maxwell's Equations

r: position (x,y,z)
t: time

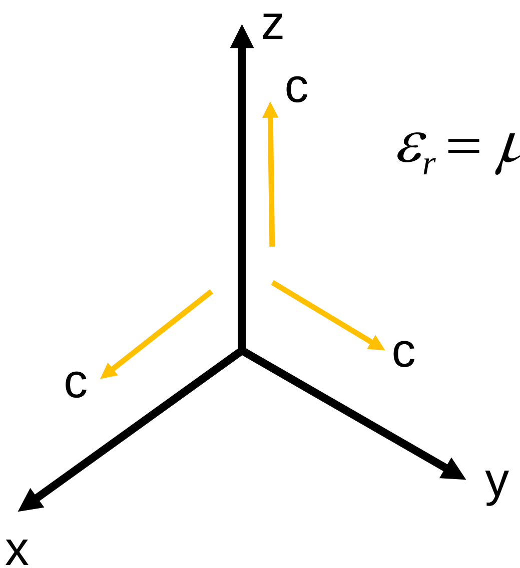
$$\vec{D}(\vec{r},t) = \varepsilon_0 \vec{E}(\vec{r},t) + \vec{p}$$
$$\vec{B}(\vec{r},t) = \mu_0 \vec{H}(\vec{r},t) + \vec{m}$$

→

$$\vec{D}(\vec{r},t) = \varepsilon_0 \varepsilon_r \vec{E}(\vec{r},t)$$
$$\vec{B}(\vec{r},t) = \mu_0 \mu_r \vec{H}(\vec{r},t)$$

: Light in bulk, homogenous, isotropic media

What is Light?



$$\varepsilon_r = \mu_r = 1$$

Simplest example: vacuum

$$\nabla \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t} - \vec{M}(\vec{r},t)$$

$$\nabla \times \vec{H}(\vec{r},t) = \frac{\partial \vec{D}(\vec{r},t)}{\partial t} + \vec{J}(\vec{r},t)$$

$$\nabla \cdot \vec{B}(\vec{r},t) = \rho_m(\vec{r},t)$$

$$\nabla \cdot \vec{D}(\vec{r},t) = \rho_e(\vec{r},t)$$

Maxwell's Equations

$\bar{F}(r,t) = Re\left[\bar{F}(r)e^{j\omega t}\right]$: Harmonic, monochromatic fields

Tip

$\nabla \times \vec{E}(\vec{r},t) \propto \partial \vec{E}(\vec{r},t)/\partial r$

$\vec{E}(\vec{r},t) = Re\left\{E_0 e^{j\omega t} e^{-j\vec{k}\cdot\vec{r}}\right\}$: E-field (V/m)

$I(\vec{r}) = E_0^2$: Intensity (V^2/m^2)

What is light?

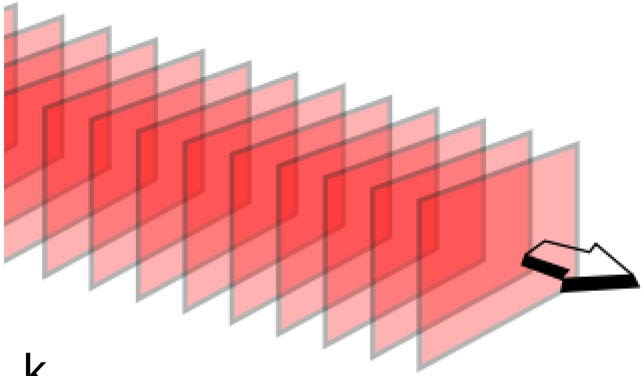
$$\vec{E}(\vec{r},t) = Re\left\{E_0 e^{j\omega t} e^{-j\vec{k}\cdot\vec{r}}\right\}$$

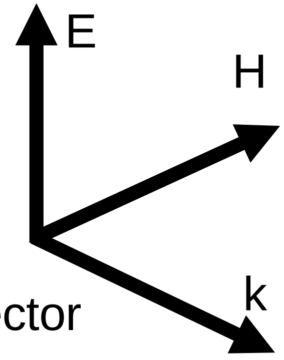
$$I(\vec{r}) = E_0^2$$

$$\vec{E}(\vec{r},t) = E_0 \cos\left(\vec{k}\cdot\vec{r} - \omega t\right)$$

$$\vec{k}\cdot\vec{r} - \omega t$$: Phase term

Plane waves (planar wavefronts)





k: wave vector

Fig. 5

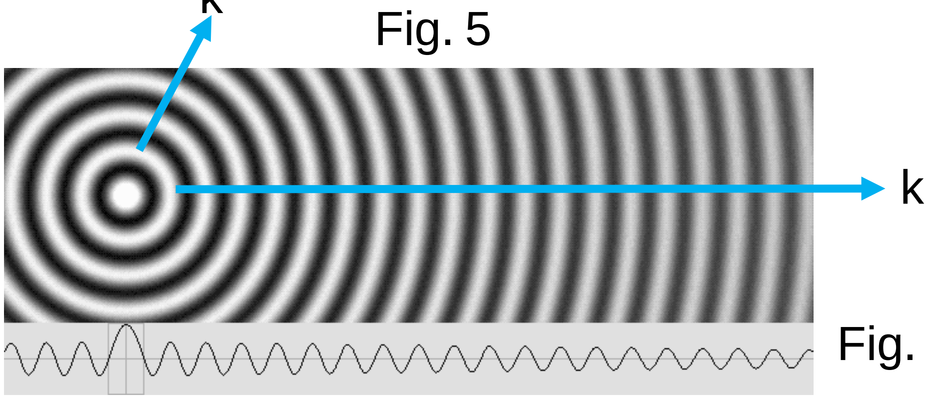


Fig. 6

- Fig. 5 original: http://upload.wikimedia.org/wikipedia/commons/2/20/Plane_wave_wavefronts_3D.svg
- Fig. 6 original: http://www.mysearch.org.uk/website3/html/12%20The%20Electron%20Phase%20Shift.htm

What is Light?

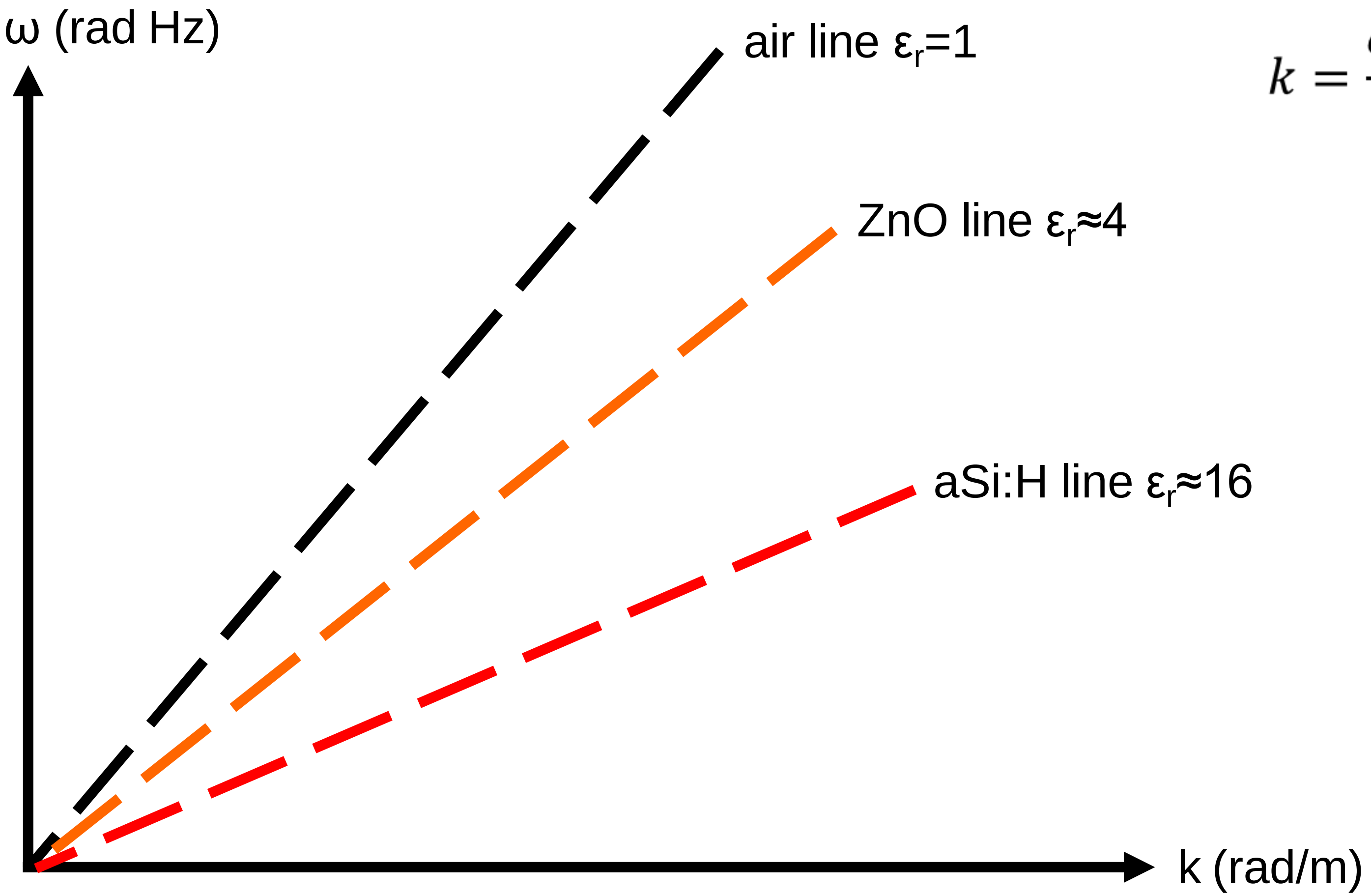
$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$: Dispersion relation (ω-k) for plane wave in free space

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8$$: speed of light in free space

$$v = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} < 3 \times 10^8$$: phase velocity in a medium with ε_r or μ_r >1

$$\sqrt{\varepsilon_r \mu_r} = n \sim \sqrt{\varepsilon_r}$$: refractive index of the medium

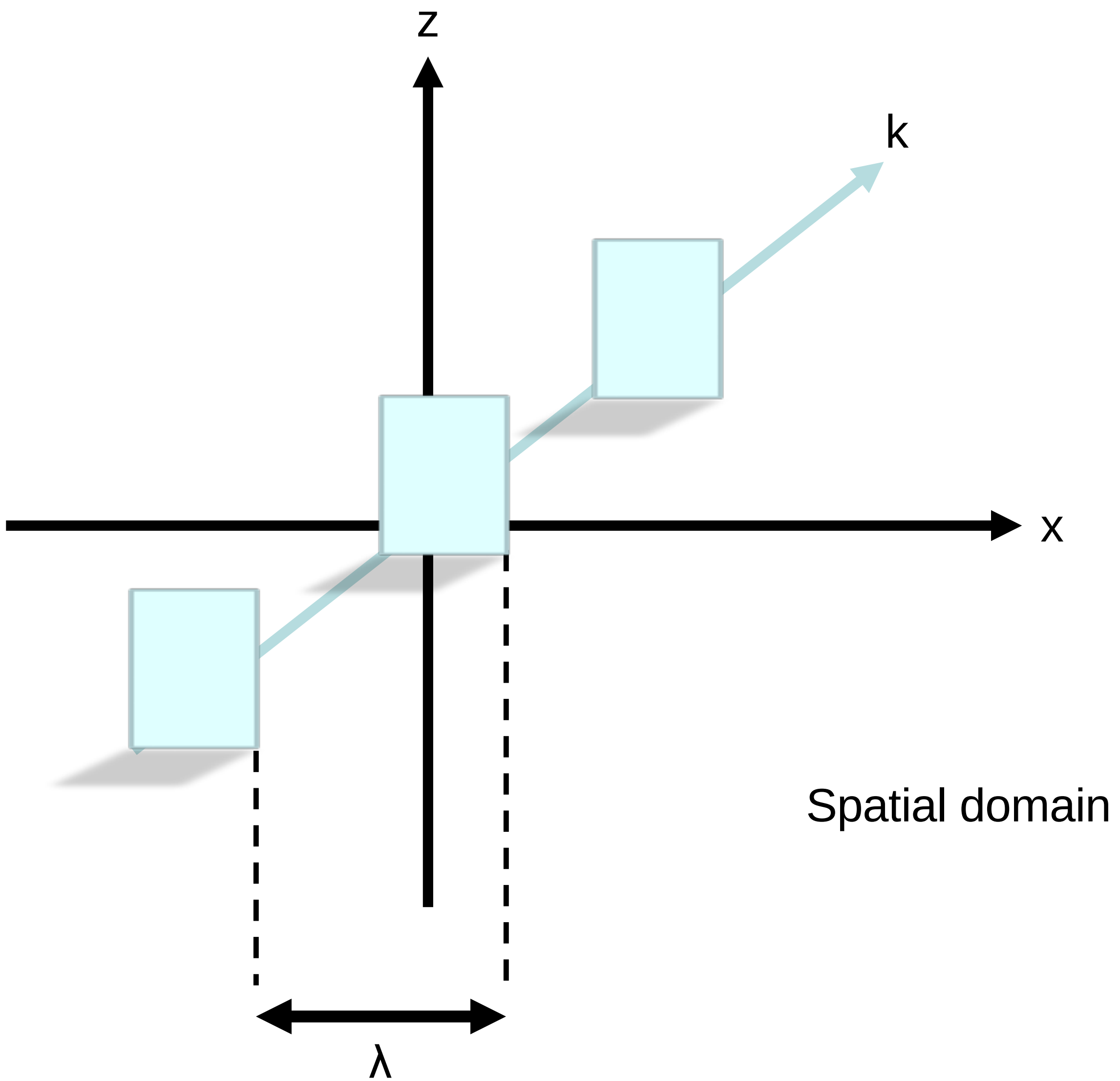
What is Light?



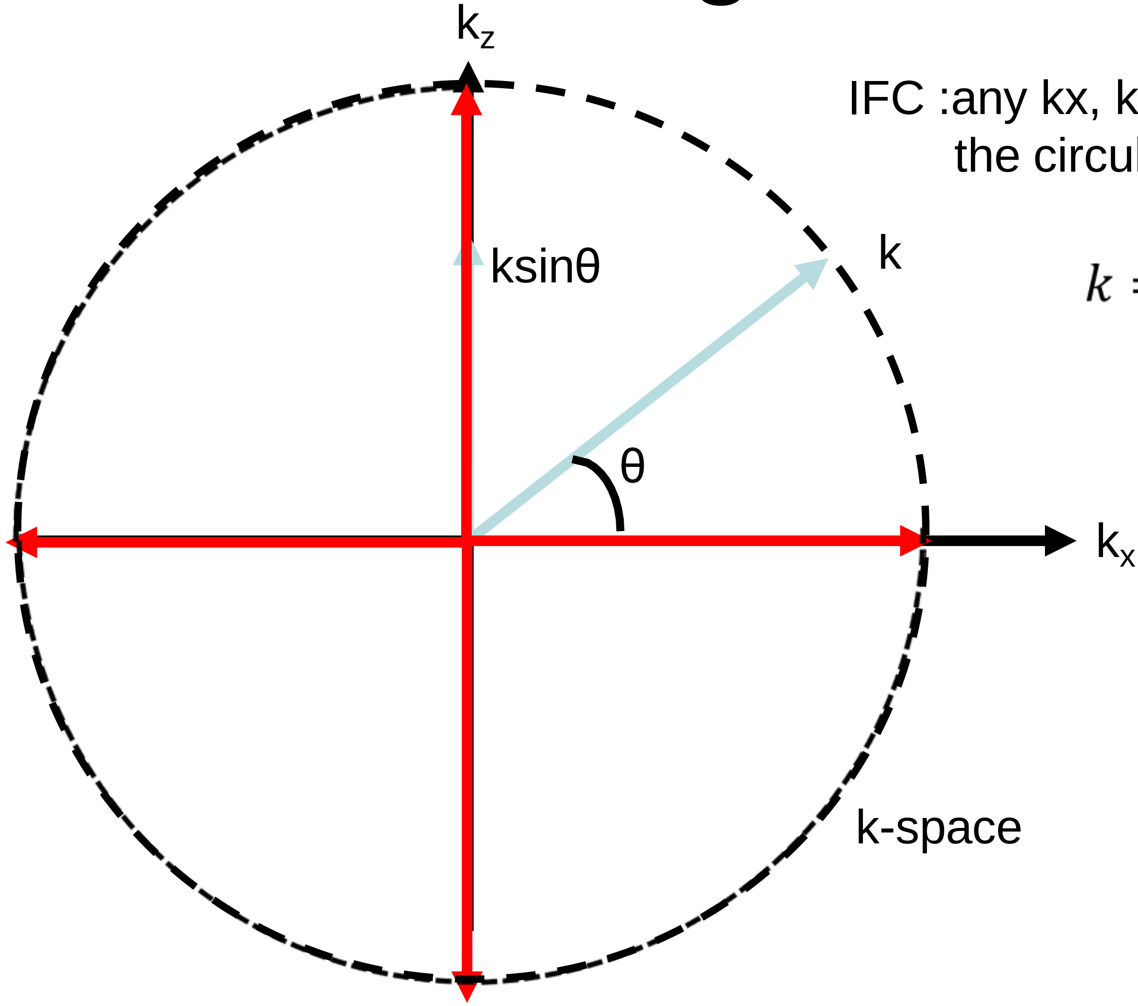
$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

Dispersion lines in bulk material

What is Light?



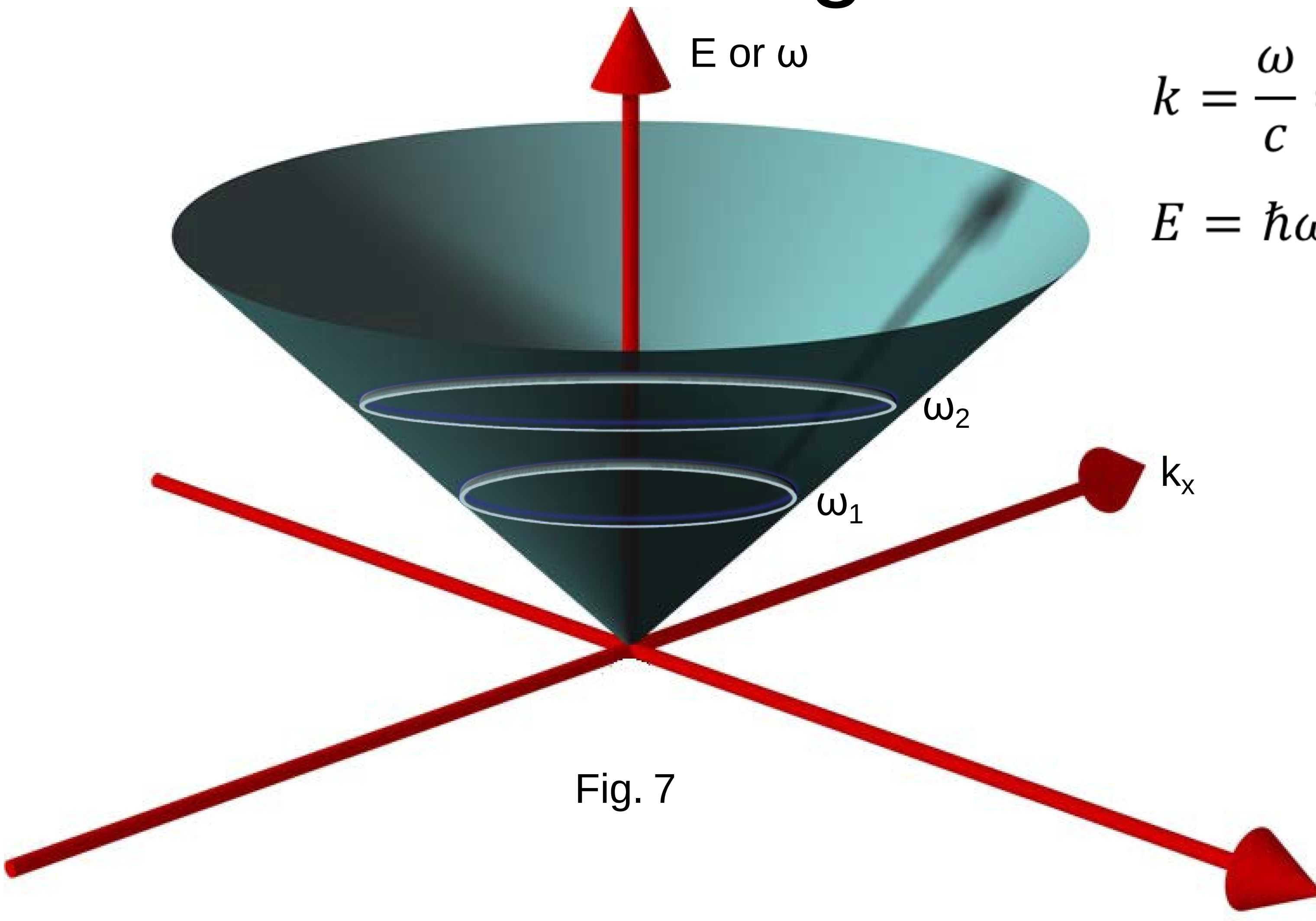
What is Light?



IFC :any kx, kz inside the circule is available

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

What is Light?



$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$E = \hbar\omega$$

Fig. 7

- Fig. 7 original: http://upload.wikimedia.org/wikipedia/commons/7/72/DoubleCone.png

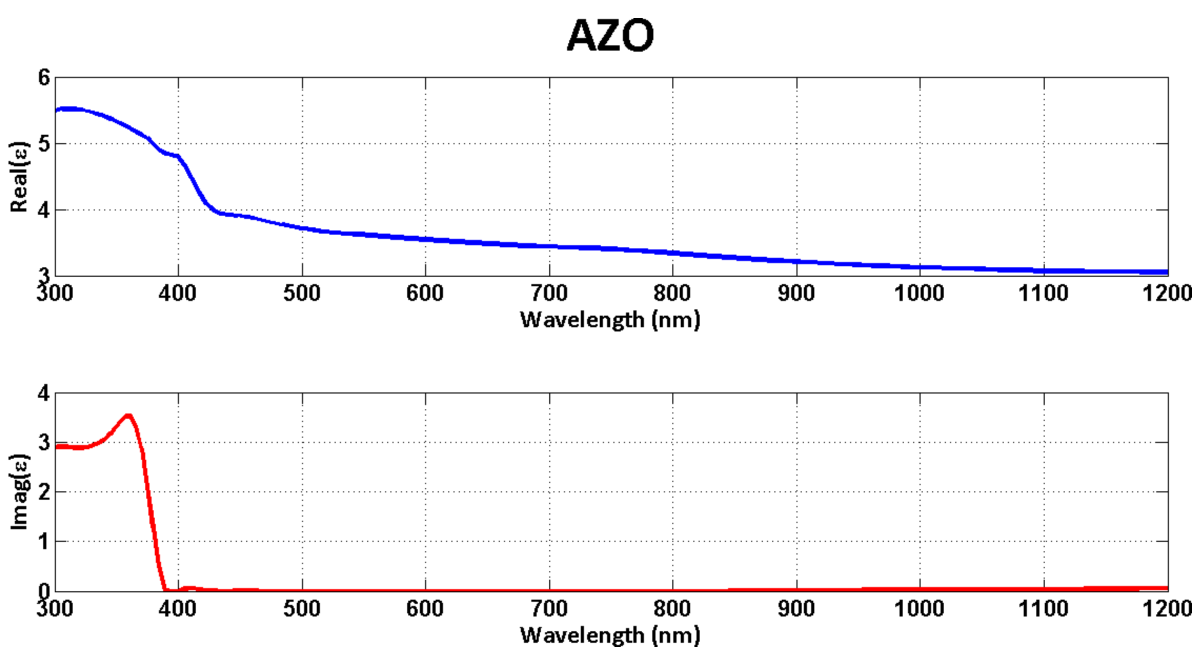
Light Absorption

$$\vec{E}(\vec{r},t) = Re\left\{E_0 e^{j\omega t} e^{-j\vec{k}\cdot\vec{r}}\right\}$$

$$I(\vec{r}) = E_0^2$$

$$k = \frac{\omega}{v} = \frac{\omega}{c} n = \frac{\omega}{c}\sqrt{\varepsilon_r}$$: What if ε_r is complex ?

$$\sqrt{\varepsilon_r} = \sqrt{\varepsilon_{real} - j\varepsilon_{imag}} = n' - jk'$$



Light Absorption

$$\vec{E}(\vec{r},t) = Re\left\{E_0 e^{j\omega t} e^{-j\vec{k}\cdot\vec{r}}\right\}$$

$$I(\vec{r}) = E_0^2$$

Let us assume propagation along x

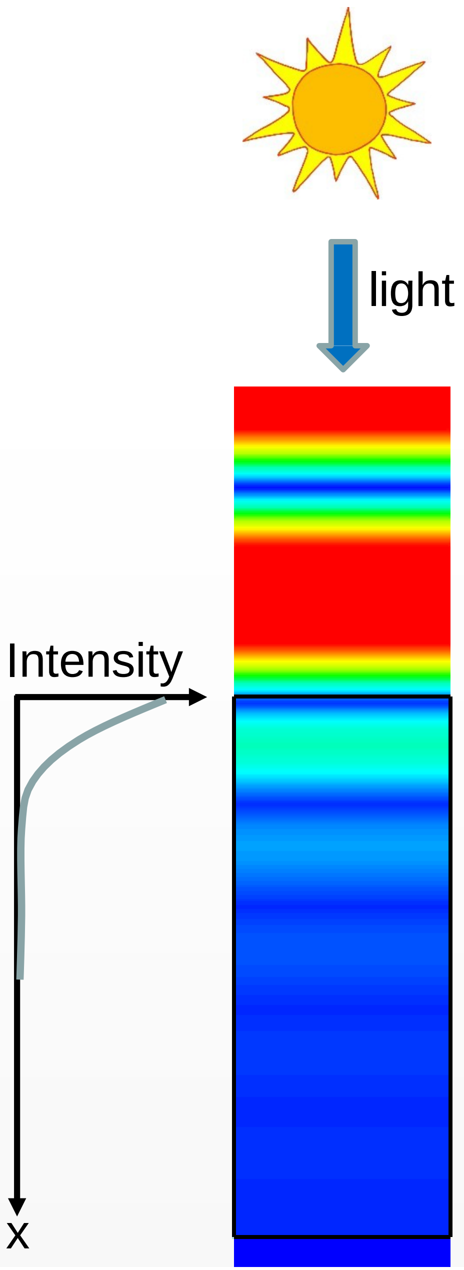
$$\vec{k}\cdot\vec{r} \Longrightarrow \frac{\omega}{c} n \times x = \frac{2\pi}{\lambda}(n' - jk')x$$

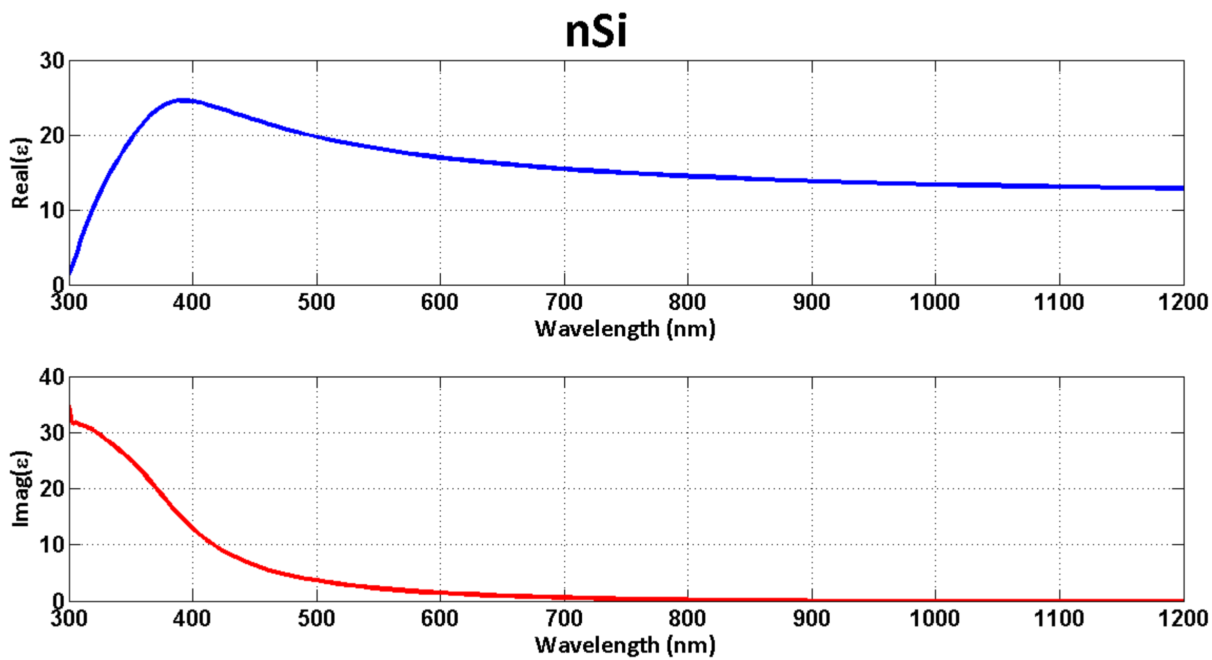
$$\vec{E}(\vec{r},t) = Re\left\{E_0 e^{j\omega t} e^{-j\vec{k}\cdot\vec{r}}\right\} = Re\left\{E_0 e^{j\omega t} e^{-j\frac{2\pi}{\lambda}(n'-jk')x}\right\}$$

$$\vec{E}(\vec{r},t) = Re\left\{E_0 e^{j\omega t} e^{-j\frac{2\pi}{\lambda}n'} e^{-\frac{2\pi}{\lambda}k'x}\right\}$$

$$I(\vec{r}) = E_0^2 e^{-\frac{4\pi}{\lambda}k'x} = E_0^2 e^{-\alpha x}$$

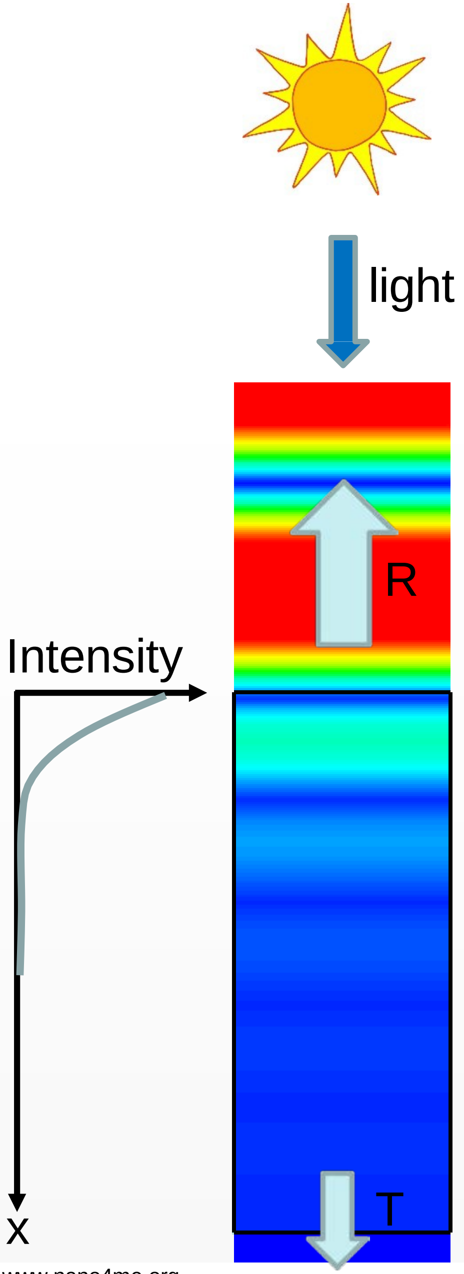
Light Absorption





$$I(x) = E_0^{\,2} e^{-\frac{4\pi}{\lambda} k' x} = E_0^{\,2} e^{-\alpha x}$$

Light Absorption



$$A(\lambda) = 1 - R(\lambda) - T(\lambda)$$

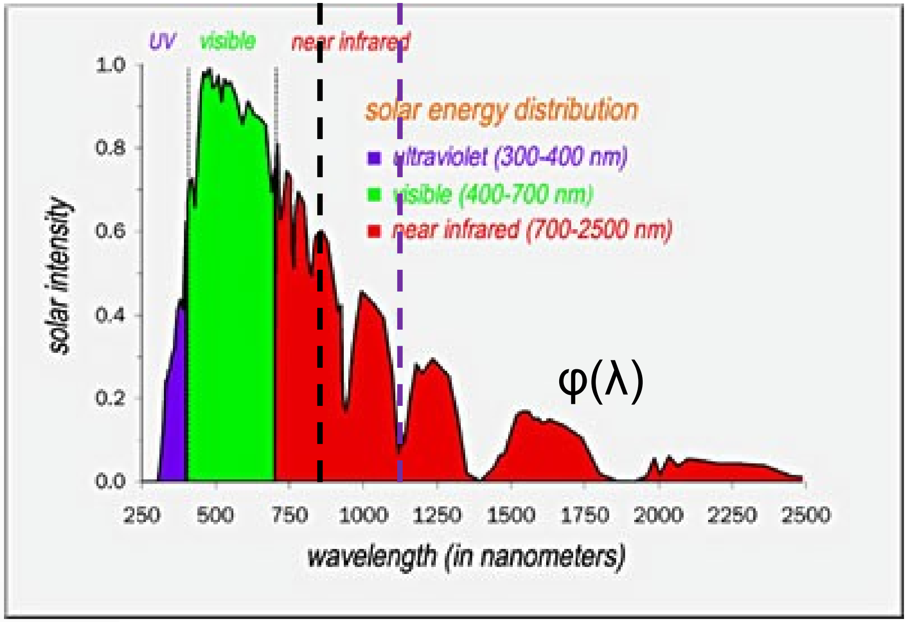


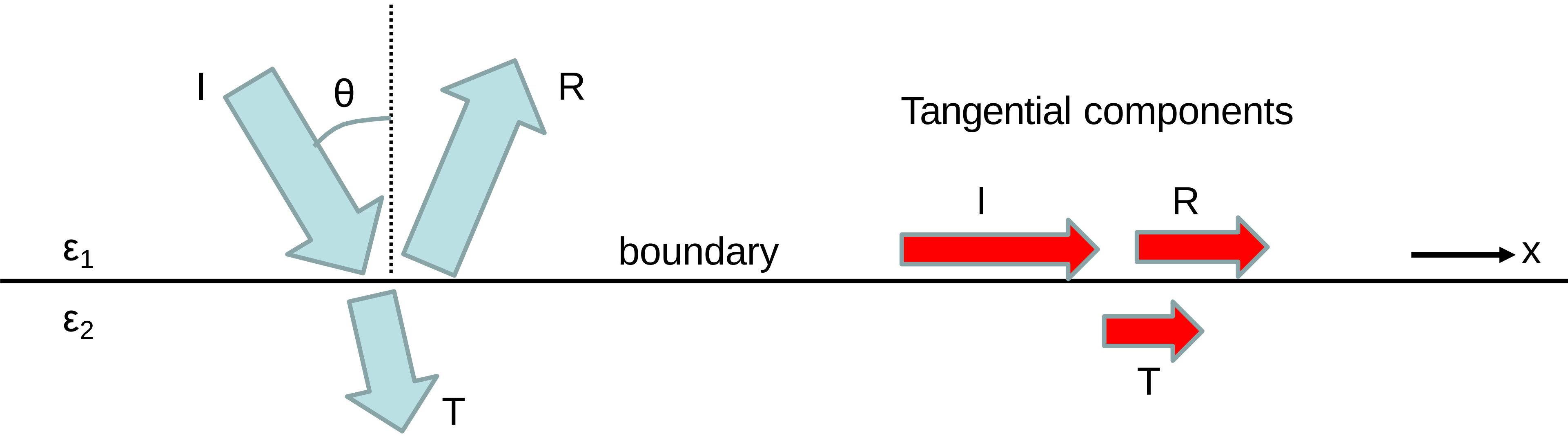
Fig. 8

- Fig. 8 taken from: http://solarjourneyusa.com/sunlight.php

$$J_{sc} \propto \int_{300nm}^{cut-off} A(\lambda)\varphi(\lambda)d\lambda$$

: short-circuit current (mA/cm^2)

Interaction of Light with Matter Reflection/Refraction



$$E_I = E_0 e^{j\omega t} e^{-jk_1 \sin\theta x}, E_R = rE_0 e^{j\omega t} e^{-jk_1 \sin\phi x}, E_T = tE_0 e^{j\omega t} e^{-jk_2 \sin\varphi x}$$

$$E_I + E_R = E_T$$

$$E_0 e^{j\omega t} e^{-jk_1 \sin\theta x} + rE_0 e^{j\omega t} e^{-jk_1 \sin\phi x} = tE_0 e^{j\omega t} e^{-jk_2 \sin\varphi x}$$

Must be satisfied for all r and t

Interaction of Light with Matter Reflection/Refraction

$$k_1 \sin\theta = k_1 \sin\phi = k_2 \sin\varphi \Rightarrow \theta = \phi$$

$$\Rightarrow \frac{\omega}{c} n_1 \sin\theta = \frac{\omega}{c} n_2 \sin\varphi \Rightarrow n_1 \sin\theta = n_2 \sin\varphi$$

Fig. 9

Fig. 10

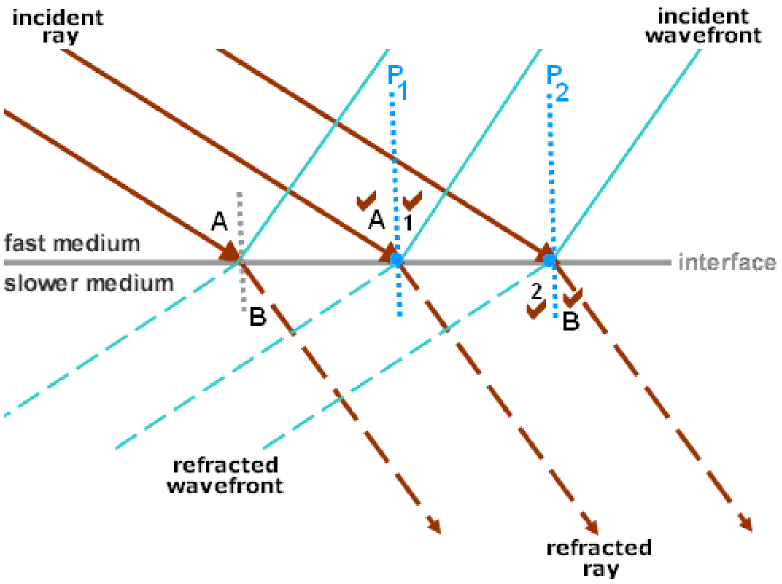


Fig. 11

- Fig. 9 taken from: http://en.wikipedia.org/wiki/Willebrord_Snellius#mediaviewer/File:Willebrord_Snellius.jpg
- Fig. 10 taken from: http://mathforum.org/mathimages/index.php/Snell's_Law
- Fig. 11 taken from: http://dev.physicslab.org/Document.aspx?doctype=3&filename=GeometricOptics_SnellsLawDerivation.xml

Interaction of Light with Matter
Reflection/Refraction

Specular Reflection	Diffuse Reflection

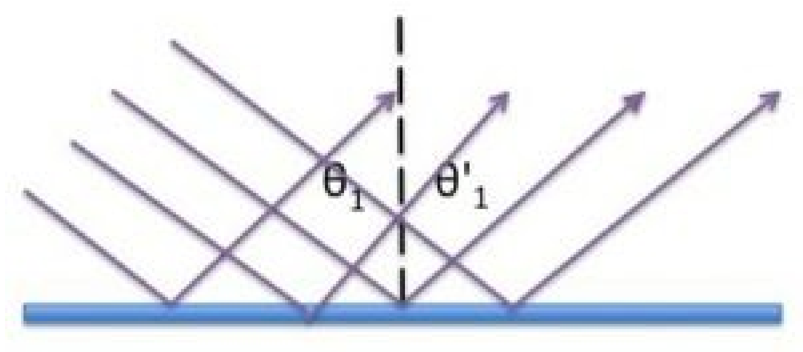


Fig. 12

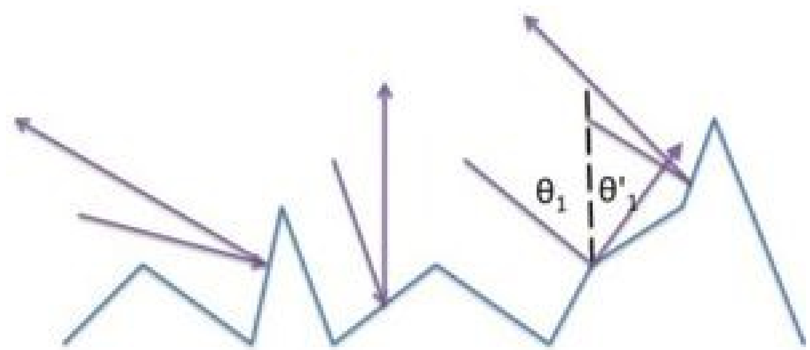
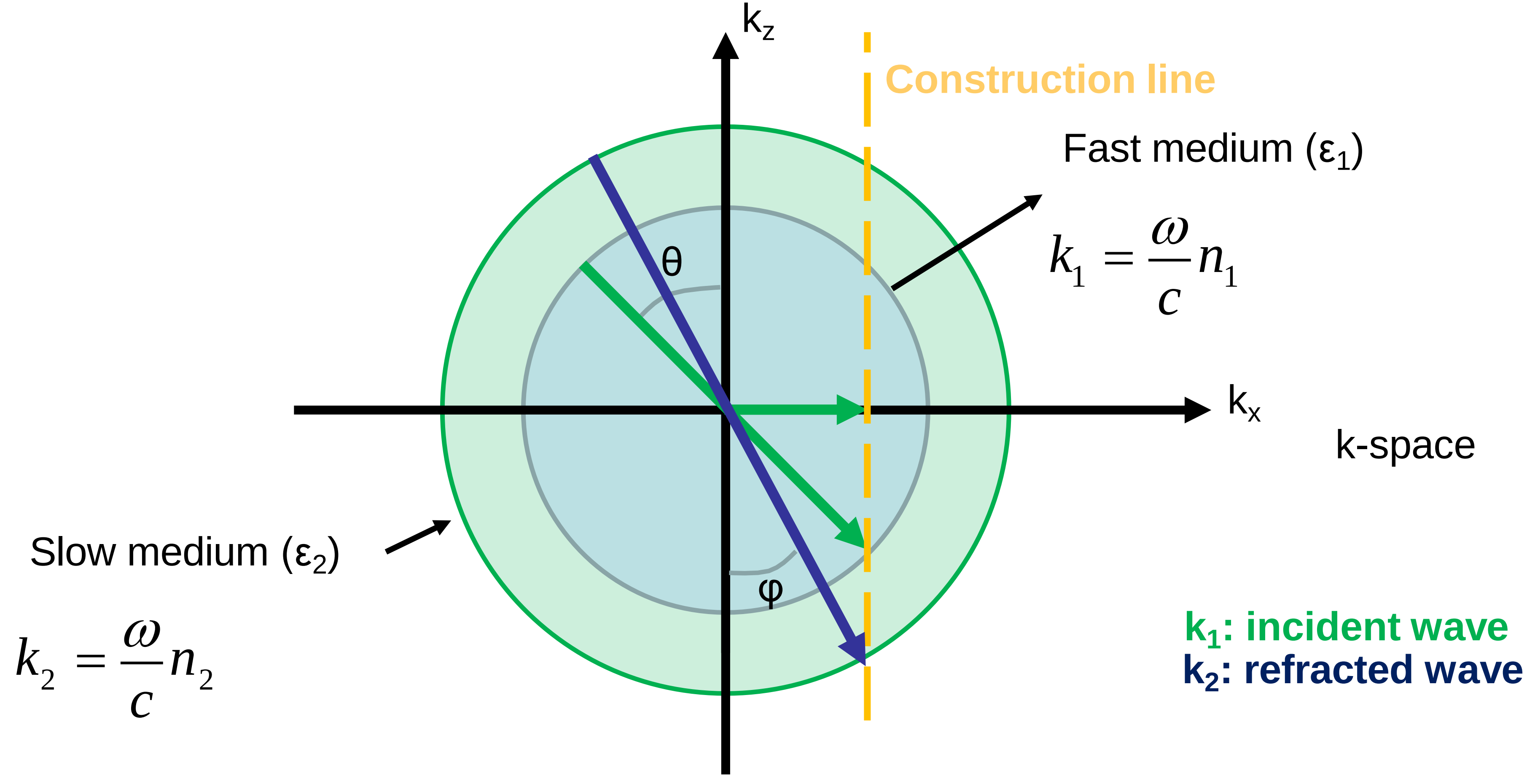


Fig. 13

- Fig. 12 and 13 taken from: http://mathforum.org/mathimages/index.php/Snell's_Law

Interaction of Light with Matter
Reflection/Refraction

Understanding Refraction with IFCs



Interaction of Light with Matter
Reflection/Refraction

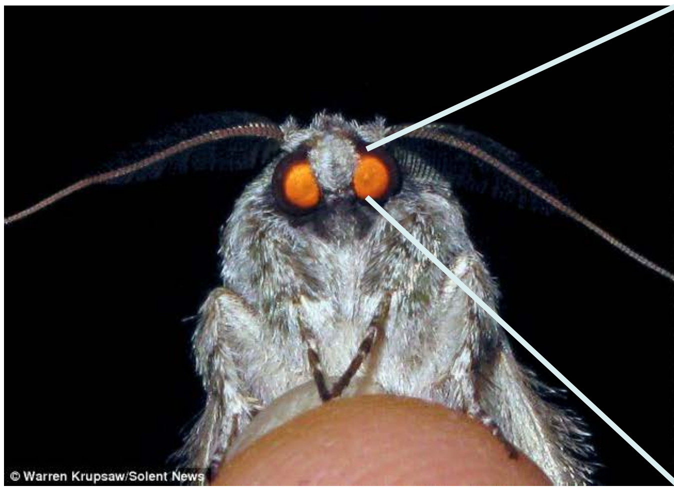


Fig. 14

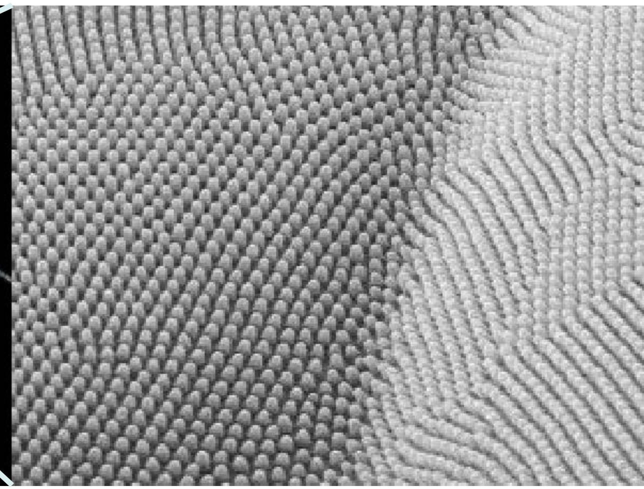


Fig. 15

Moth's eye under SEM

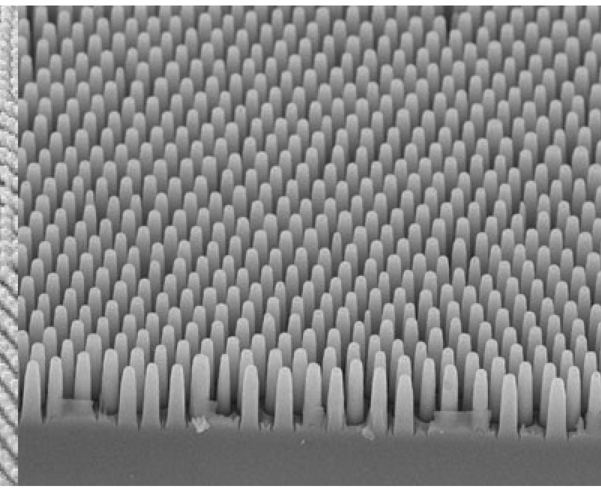


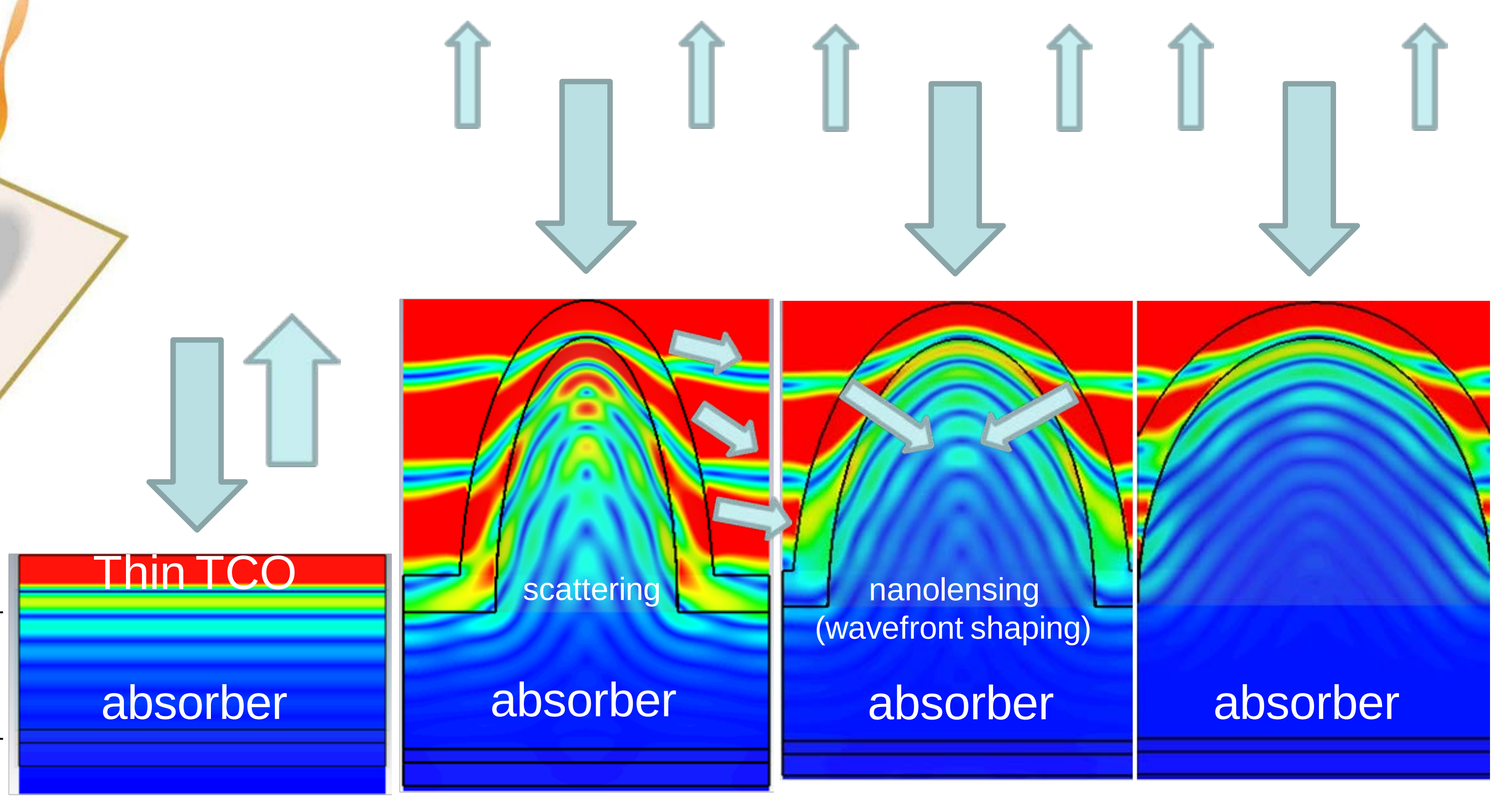
Fig. 16

Designs inspired from moth's eye

- Fig. 14 taken from: http://www.dailymail.co.uk/news/article-2195134/Americas-MOTH-model--Winged-insects-pose-tip-finger-stunning-close-ups.html
- Fig 15 and 16 taken from: http://www.nanowerk.com/spotlight/spotid=7938.php

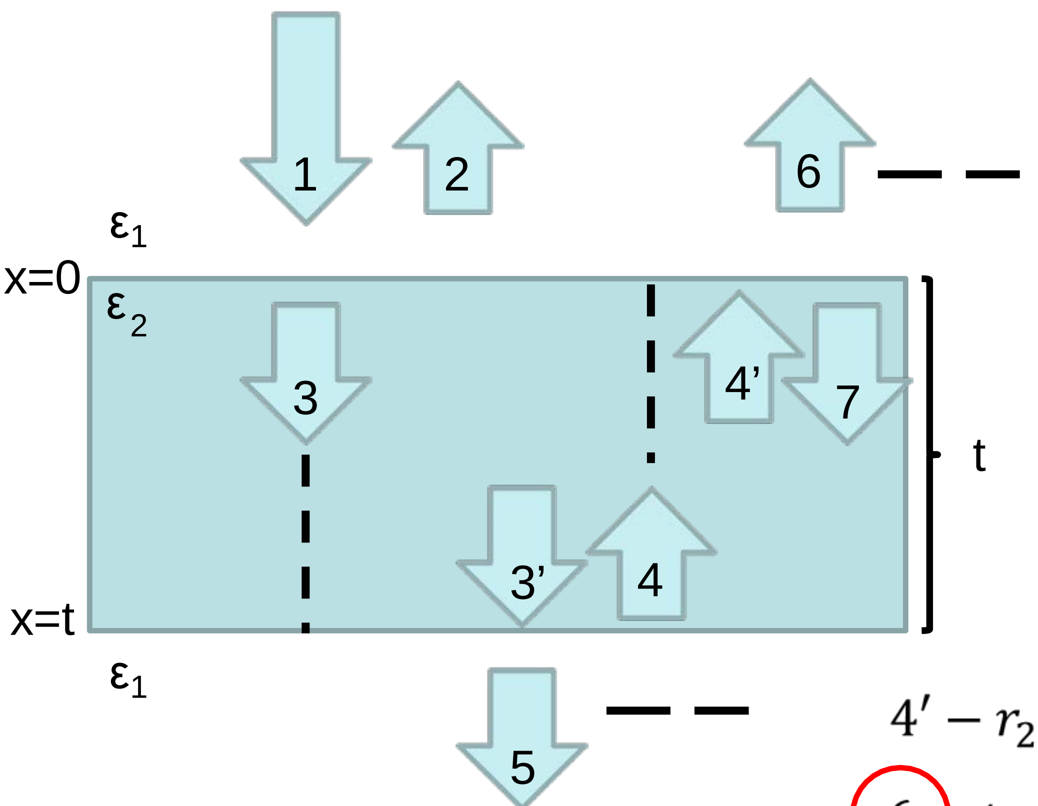
Interaction of Light with Matter
Reflection/Refraction

Fig. 17



- Fig. 17 taken from: http://cnx.org/contents/031da8d3-b525-429c-80cf-6c8ed997733a@8.8:204/College_Physics

Interaction of Light with Matter
Fabry Perot Resonances



$$1 - E_0 e^{-jk_1 x} \xrightarrow{x=0} E_0$$

$$2 - r_1 E_0 e^{+jk_1 x} \xrightarrow{x=0} r_1 E_0$$

$$3 - t_1 E_0 e^{-jk_2 x} \xrightarrow{x=0} t_1 E_0$$

$$3' - t_1 E_0 \xrightarrow{t} t_1 E_0 e^{-jk_2 t}$$

$$4 - r_2 t_1 E_0 e^{-jk_2 t}$$

$$5 - t_2 t_1 E_0 e^{-jk_2 t}$$

$$4' - r_2 t_1 E_0 e^{-jk_2 t} \xrightarrow{-t} r_2 t_1 E_0 e^{-jk_2 t} e^{+jk_2(-t)}$$

$$6 - t_2 r_2 t_1 E_0 e^{-2jk_2 t}$$

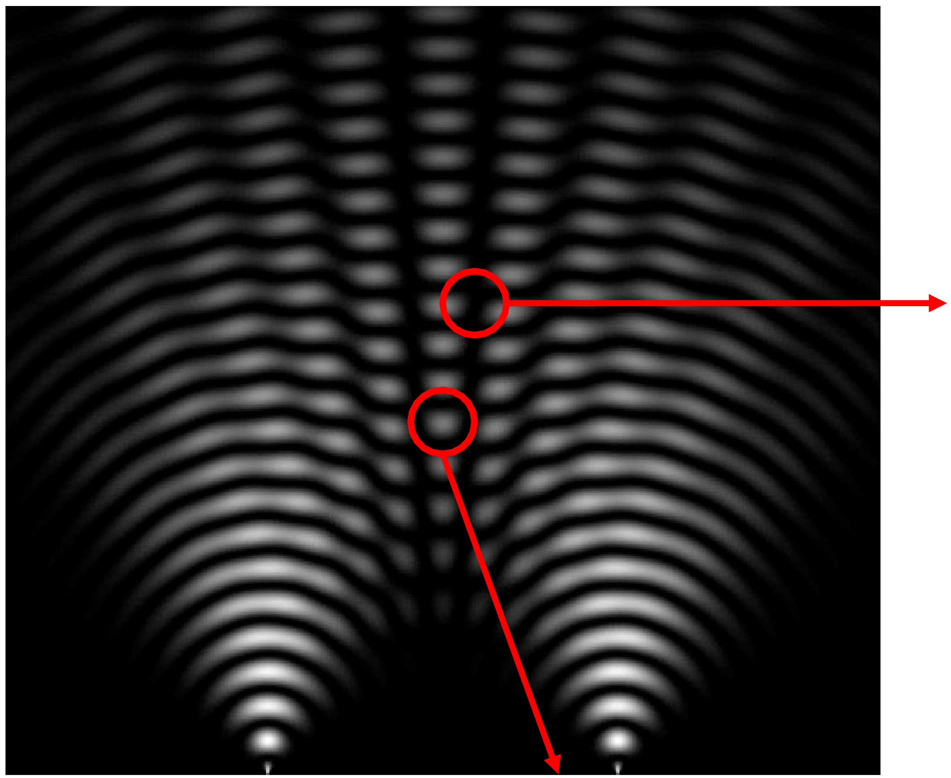
$$7 - r_2 r_2 t_1 E_0 e^{-2jk_2 t}$$

Total R: 2+6+…

Interaction of Light with Matter
Fabry Perot Resonances

Interference

$$R_{x=0} = r_1E_0 + t_2r_2t_1E_0e^{-2jk_2t} + t_2r_2r_2r_2t_1E_0e^{-4jk_2t} + \square$$



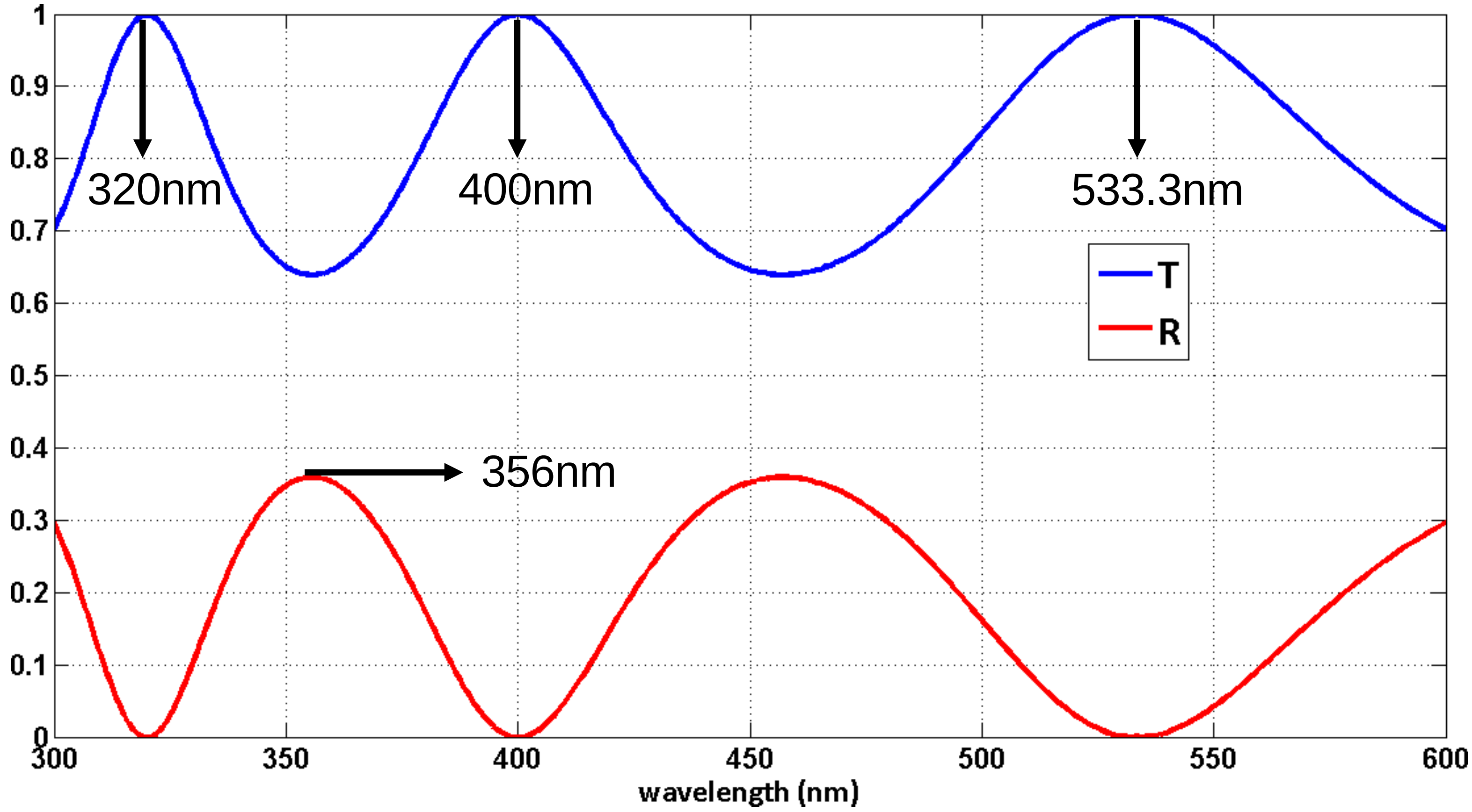
Dark spots

Bright areas

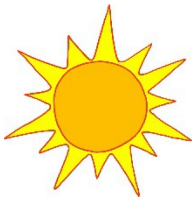
Fig. 18

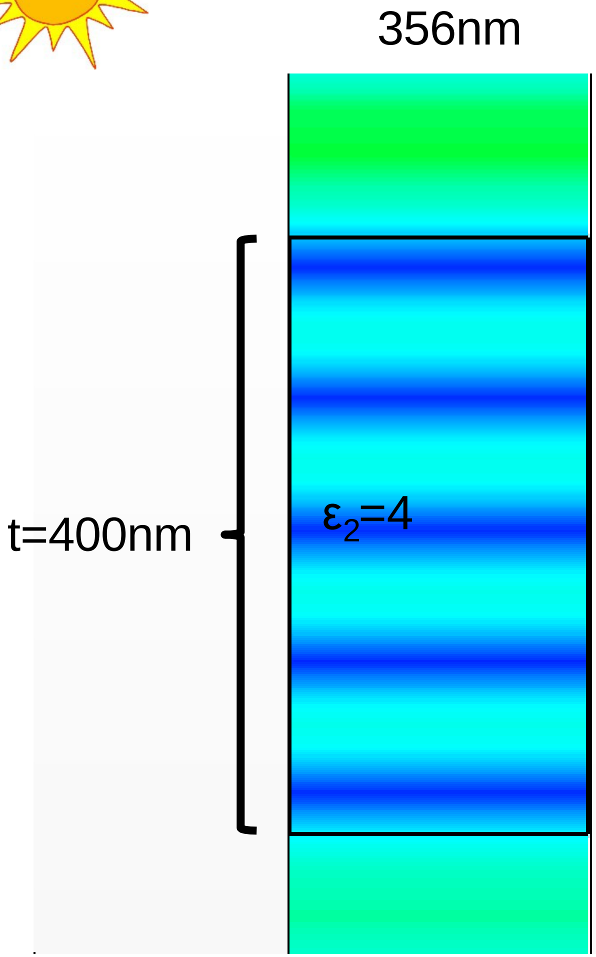
- Fig. 18 original: http://emfsafetynetwork.org/maine-utility-admits-smart-meters-cause-interference/

Interaction of Light with Matter
Fabry Perot Resonances



Interaction of Light with Matter
Fabry Perot Resonances



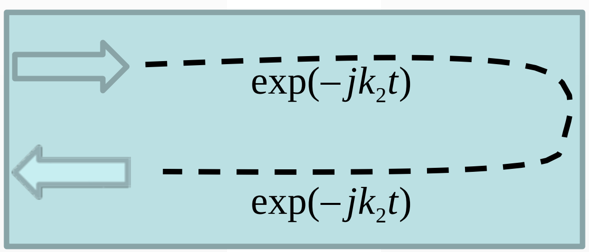


No reflection

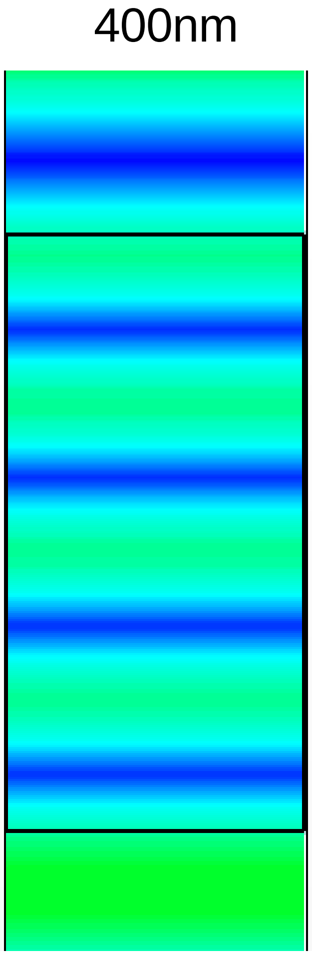
$$2k_2t = 2\pi m \Rightarrow \lambda m = 2n_2t$$

$$m \Rightarrow \text{integer, eg. m=0,1}$$

Standing wave formation

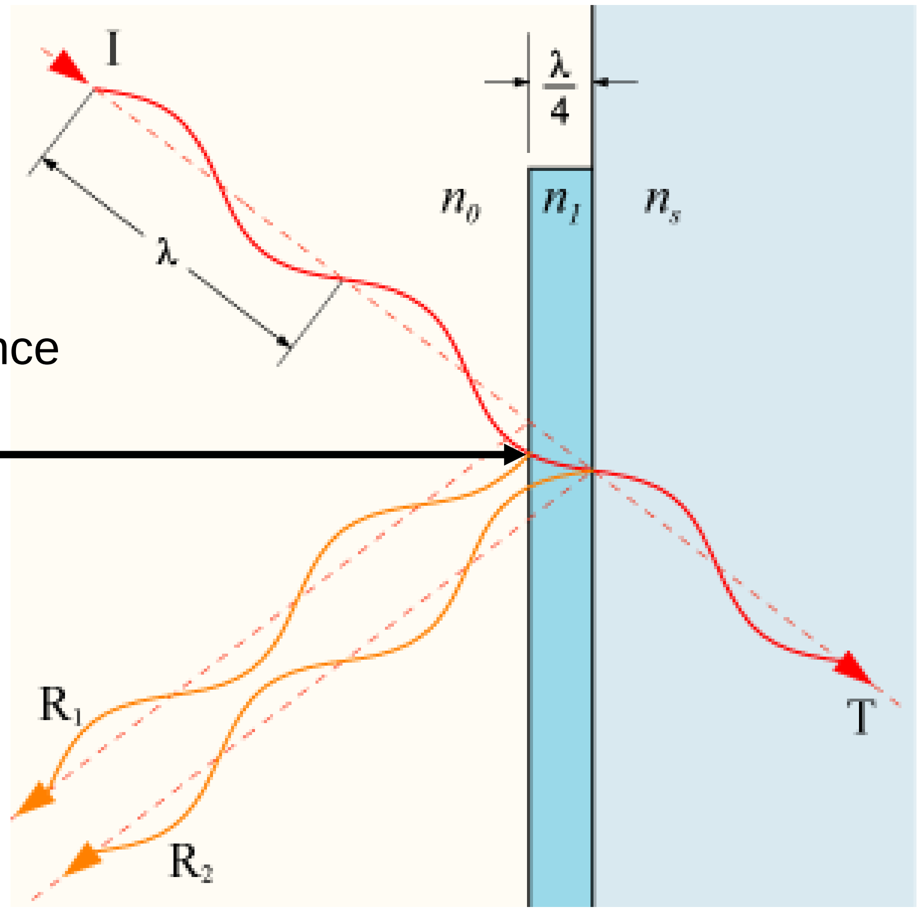


x



Interaction of Light with Matter
Fabry Perot Resonances

Destructive interference at this boundary



Quarter wavelength matching

TCO

Fig. 19

- Fig. 19 original: http://upload.wikimedia.org/wikipedia/commons/8/8c/Optical-coating-2.png/

Interaction of Light with Matter
Fabry Perot Resonances

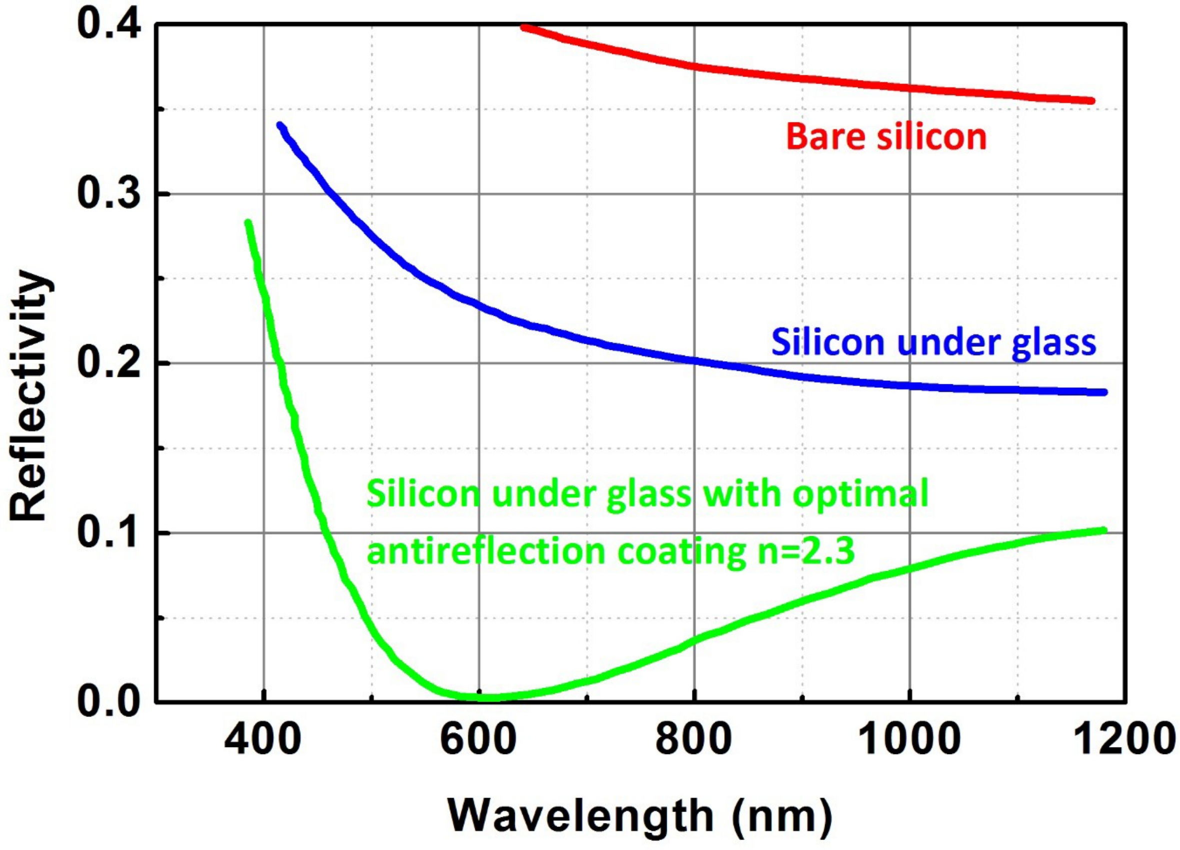


Fig. 20

- Fig. 20 recreated with the data collected from: http://pveducation.org/pvcdrom/design/anti-reflection-coatings

Interaction of Light with Matter
Waveguiding

light

Light ?

Standing Waves inside

slab

Light guiding

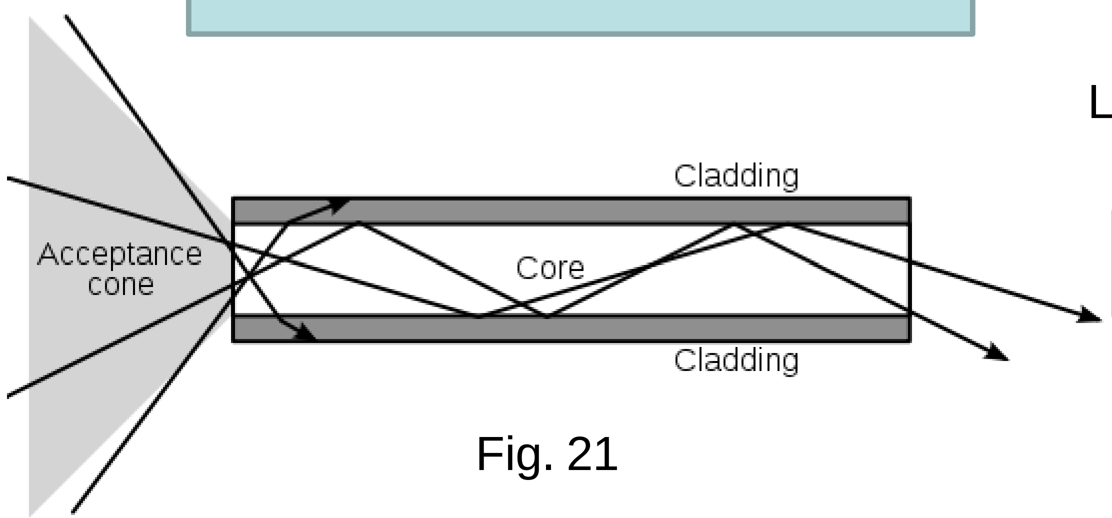


Fig. 21

- Fig. 21 taken from: http://photonicswiki.org/index.php?title=File:Optical-fibre.svg

Interaction of Light with Matter
Waveguiding

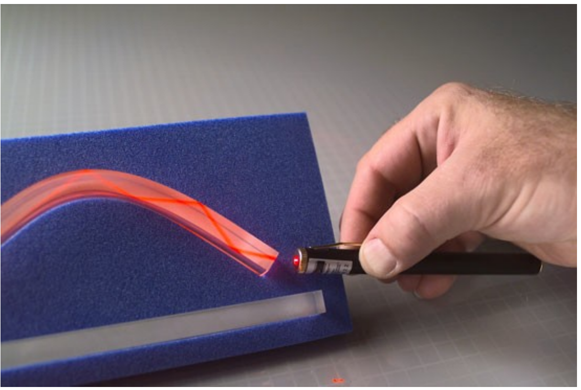
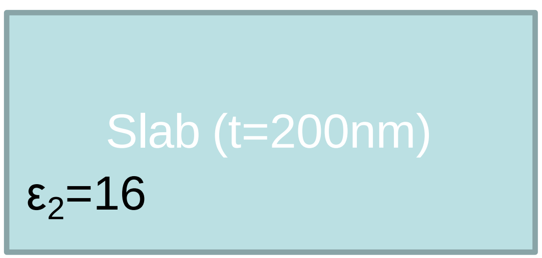
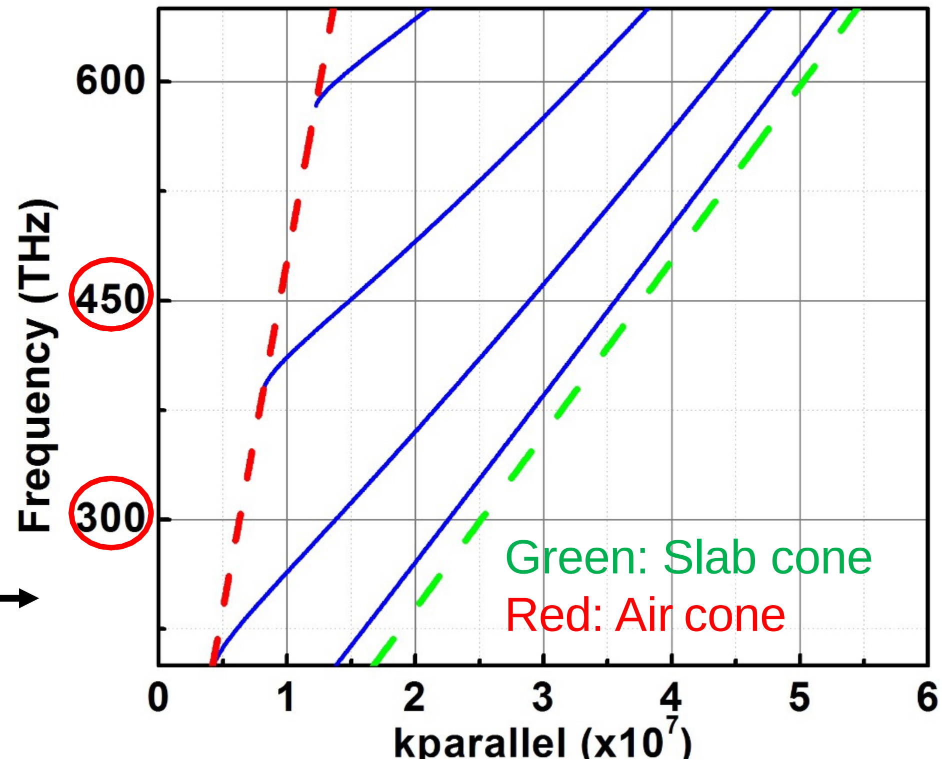


Fig. 22

$\varepsilon_1=1$



$\varepsilon_1=1$

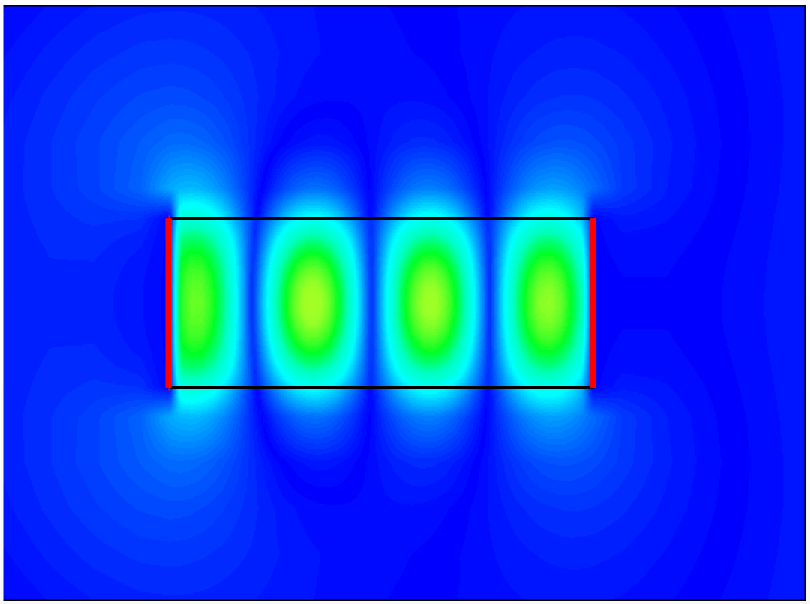


Dispersion results for a dielectric waveguide

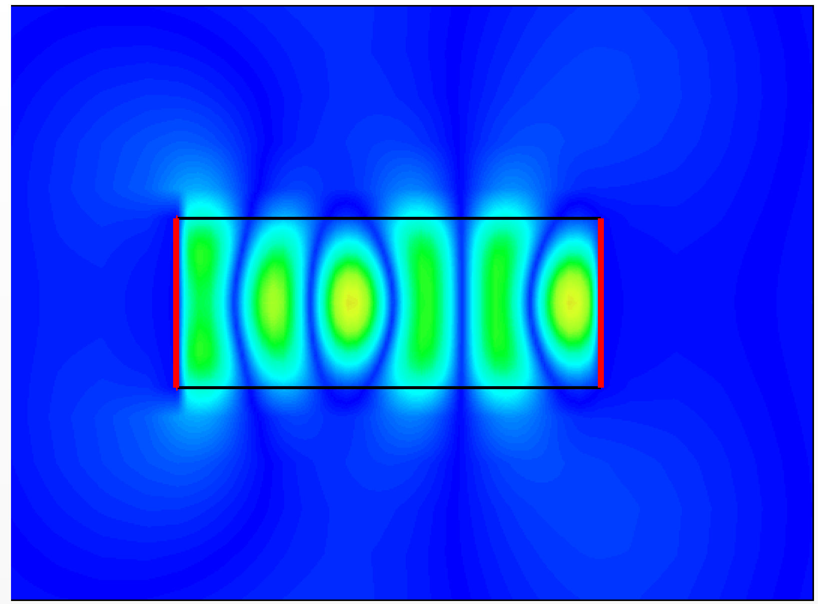
- Fig. 22 taken from: http://i-fiberoptics.com/images/if-547-lg.jpg

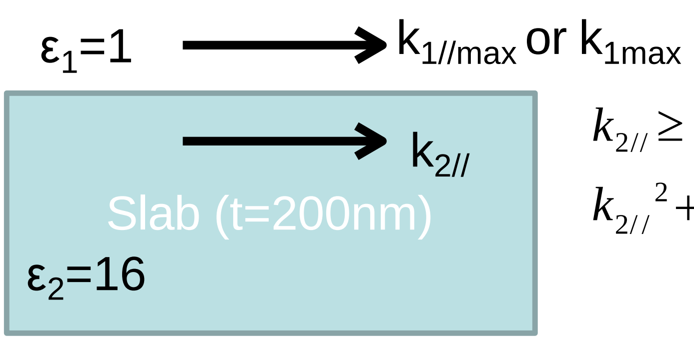
Interaction of Light with Matter
Waveguiding

300 THz



450 THz

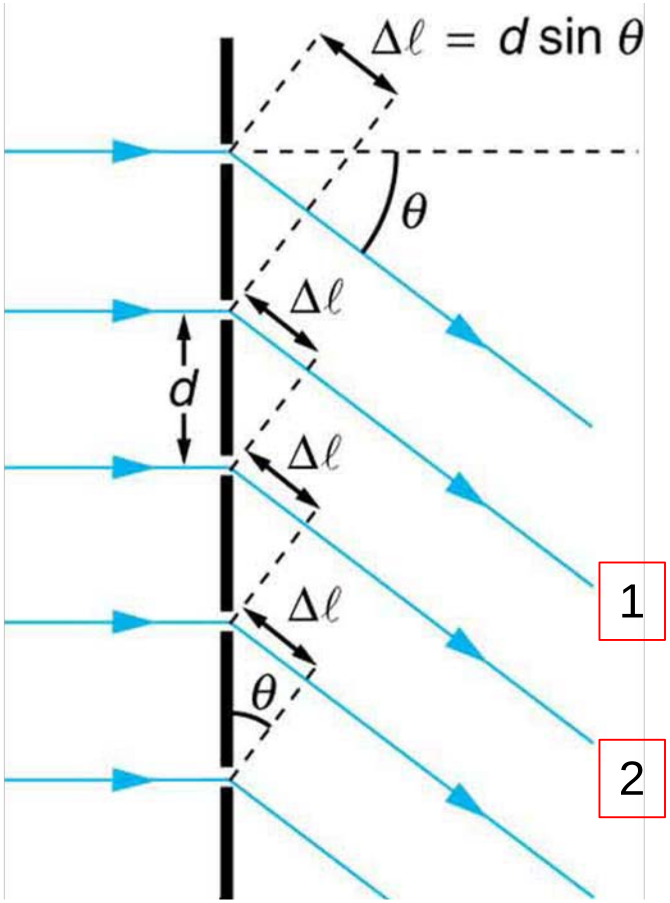




$$k_{2//} \ge k_{1//\,max}(k_{1z}=0) \qquad k_{2//}=k_{1//}$$

$$k_{2//}^2 + k_{2z}^2 = k_2^2 \qquad k_{1//\,max}^2 - k_{2//}^2 = k_{1z}^2 \Rightarrow k_{1z} : \text{imaginary}$$

Interaction of Light with Matter
Diffraction



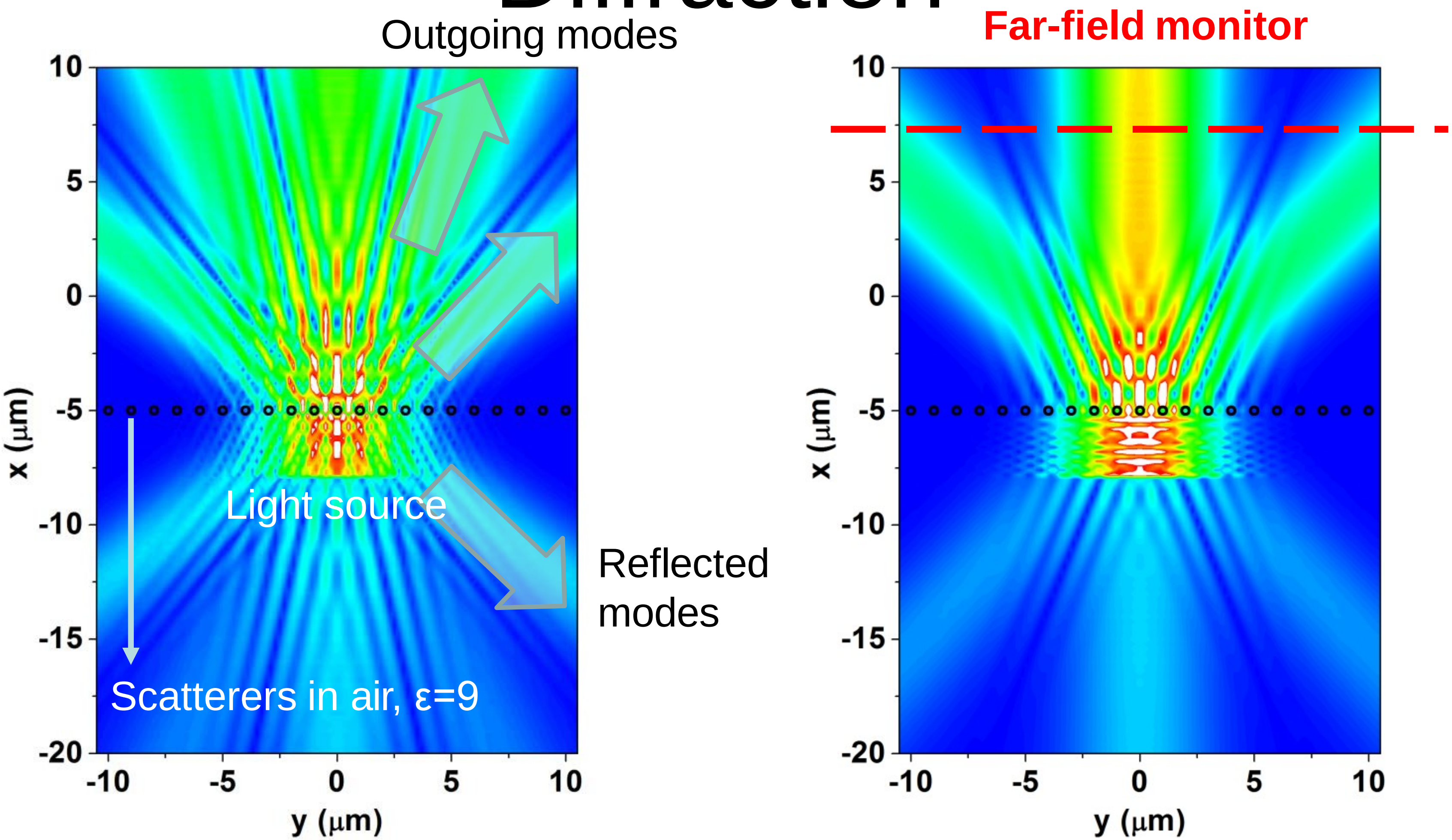
$$\phi_1 - \phi_2 = \nabla\phi$$

$$\nabla\phi = k\Delta\square = \frac{2\pi}{\lambda}\Delta\square = 2\pi m$$

$$\Delta\square = d\sin\theta \Rightarrow \sin\theta = \frac{m\lambda}{d}$$

A collective action as soon as there is discontinuity along the reflection plane.

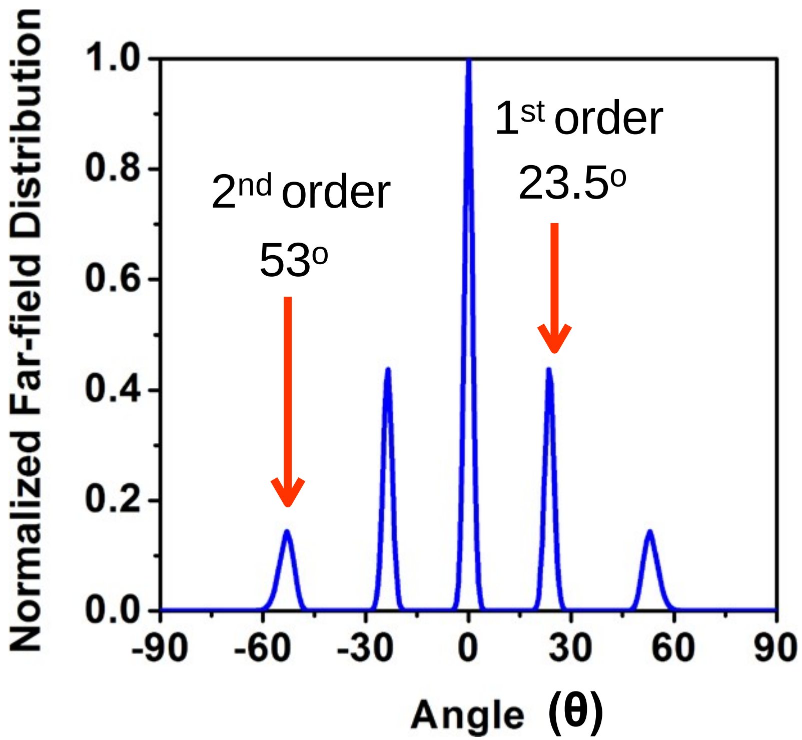
Interaction of Light with Matter
Diffraction

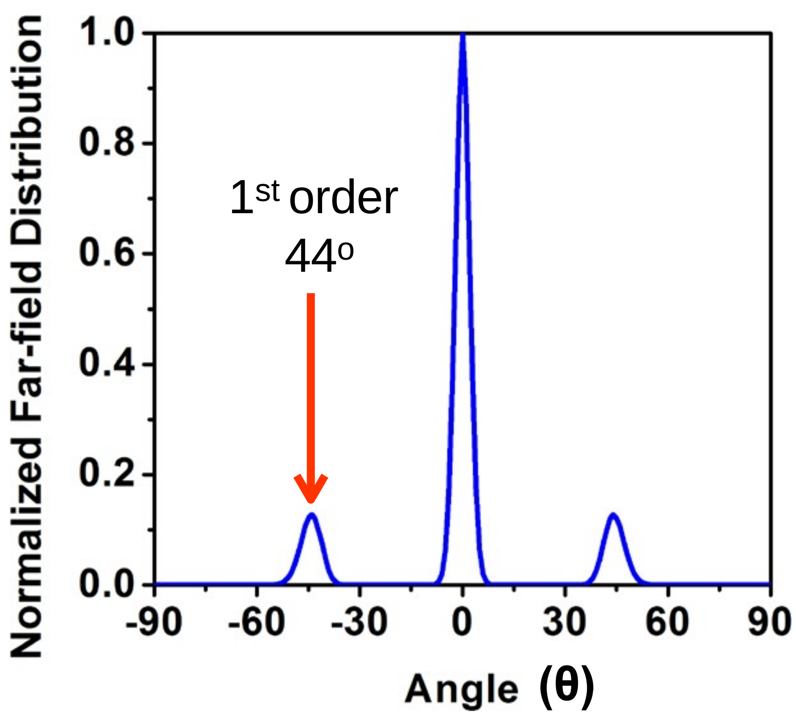


λ=400 nm
d=1000 nm

λ=700 nm
d=1000 nm

Interaction of Light with Matter
Diffraction





The higher order diffractions (1st, 2nd , etc.…) are observed at angles that obey the simple formulation

λ=400 nm

d=1000 nm

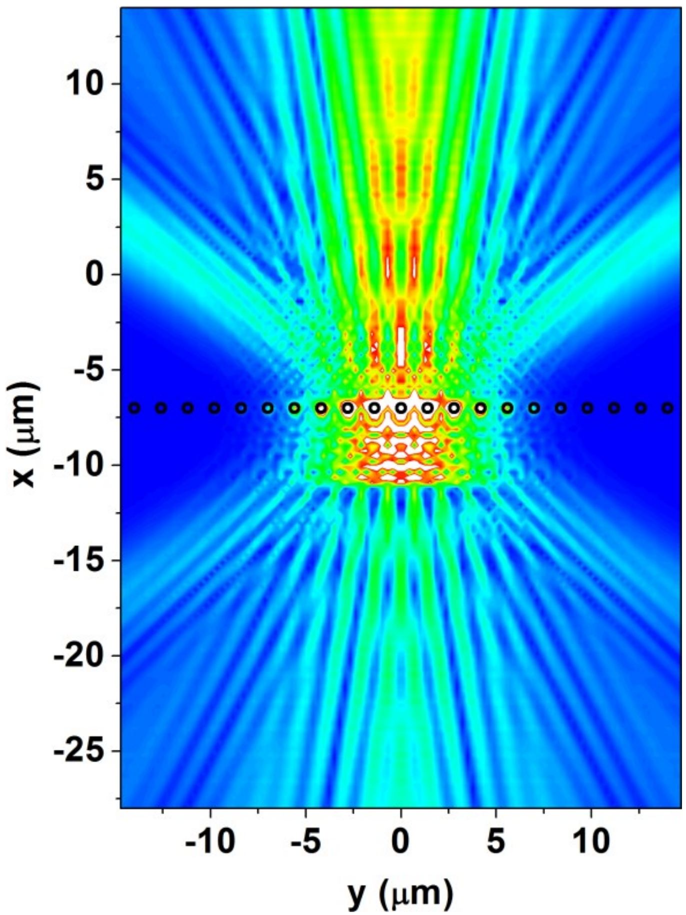
e.g. $\sin^{-1}(700 / 1000) = 44.42^{o}$ for +1st order diffraction

λ=700 nm

d=1000 nm

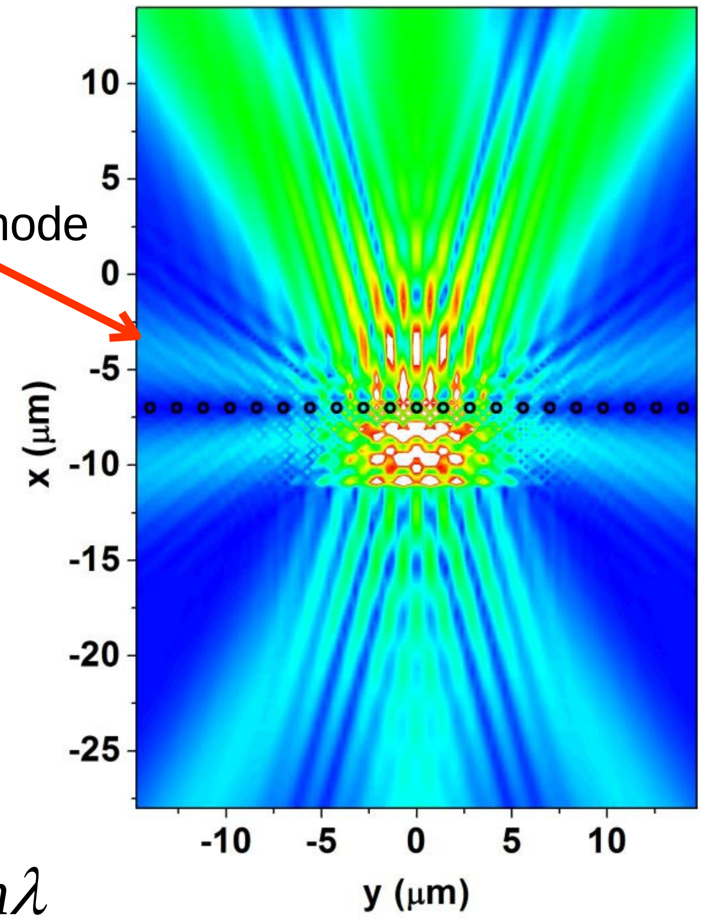
**** Notice the small deviations (less than 1^{o}) between the simulation results and diffraction formula**

Interaction of Light with Matter
Diffraction



λ=400 nm
d=1400 nm

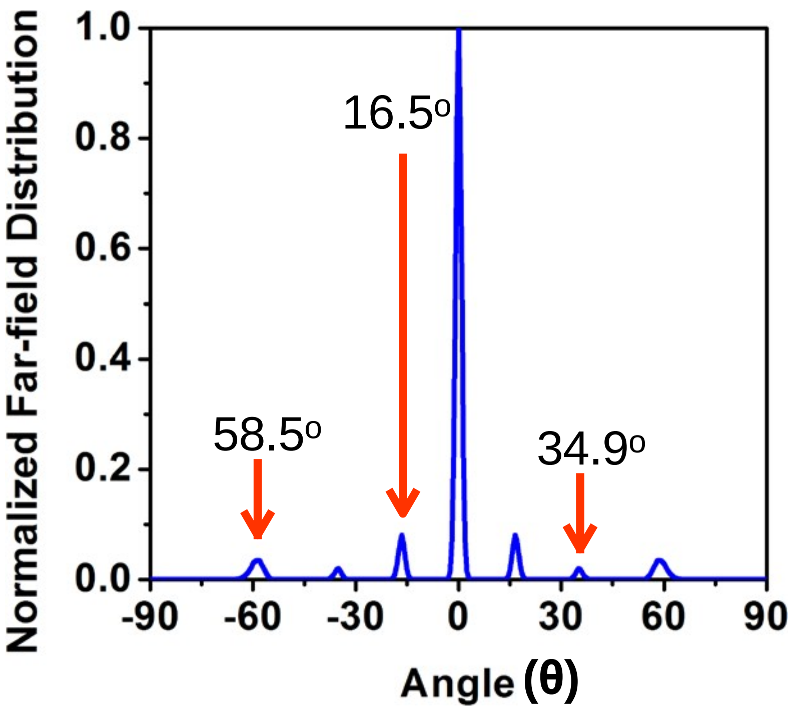
Tangential mode

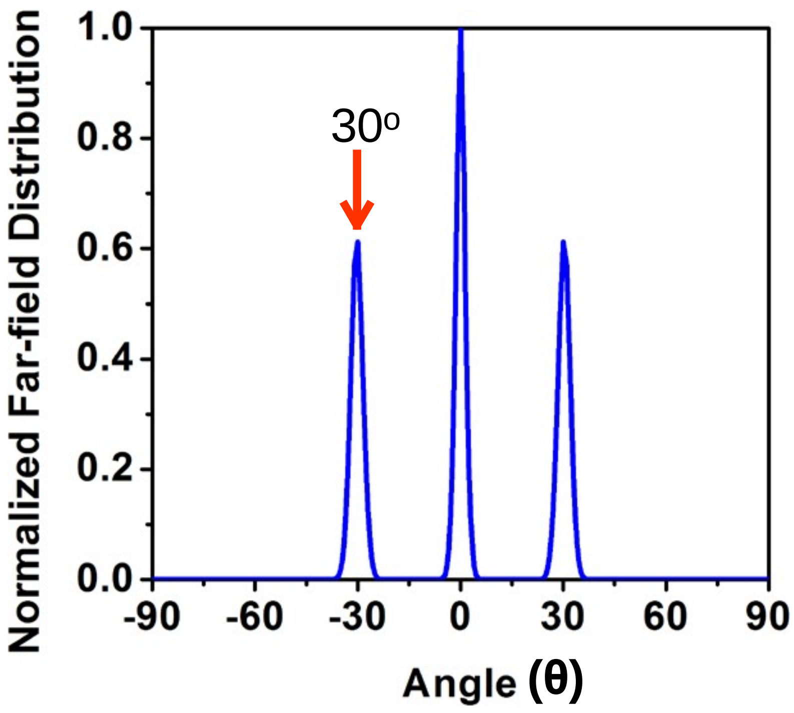


λ=700 nm
d=1400 nm

$$\sin\theta = \frac{m\lambda}{d}$$

Interaction of Light with Matter
Diffraction

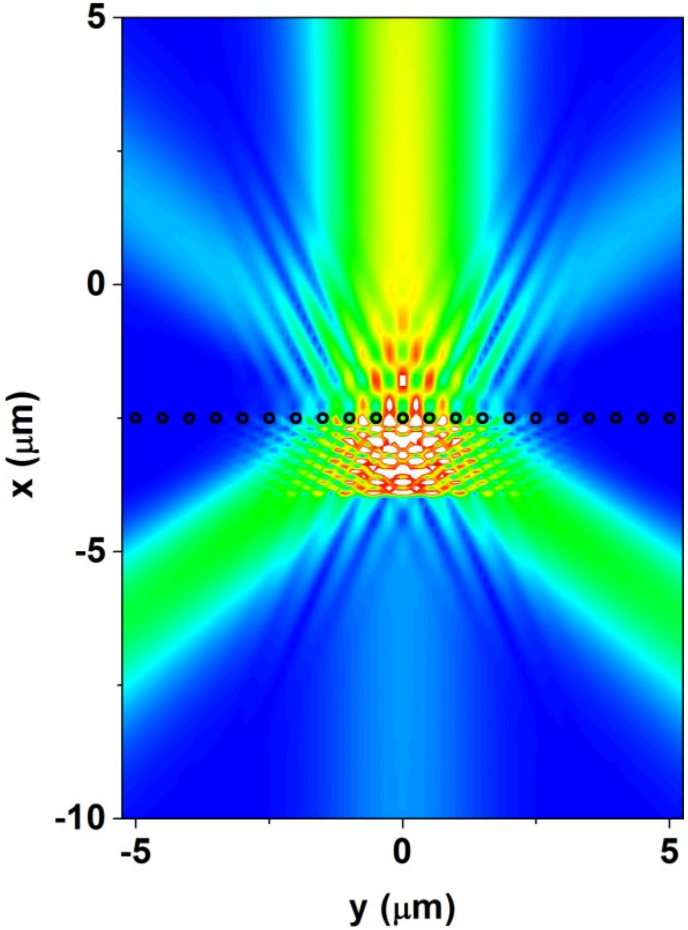




λ=400 nm
d=1400 nm

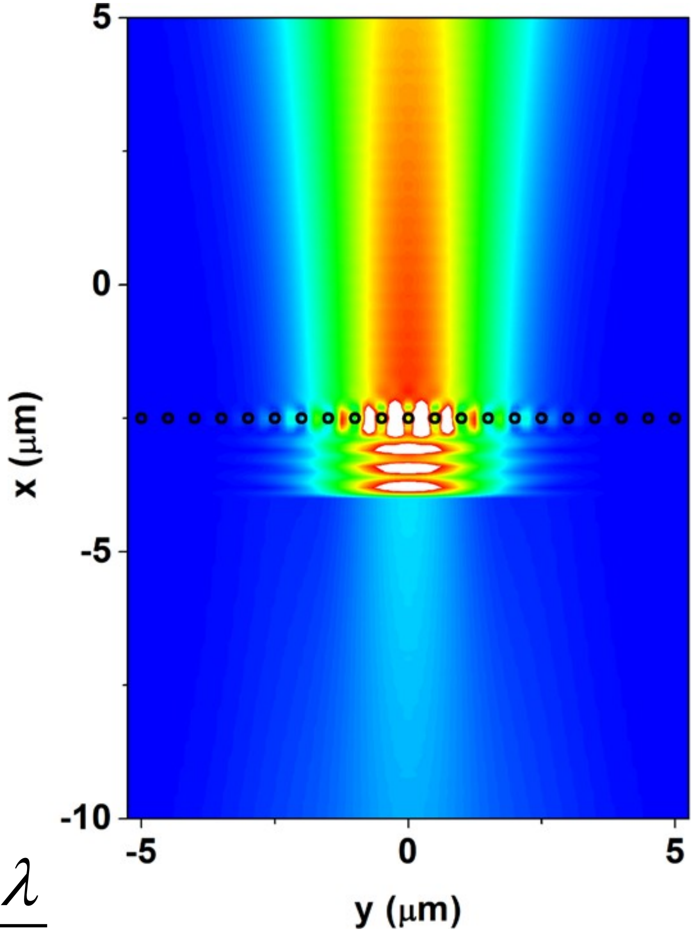
λ=700 nm
d=1400 nm

Interaction of Light with Matter
Diffraction



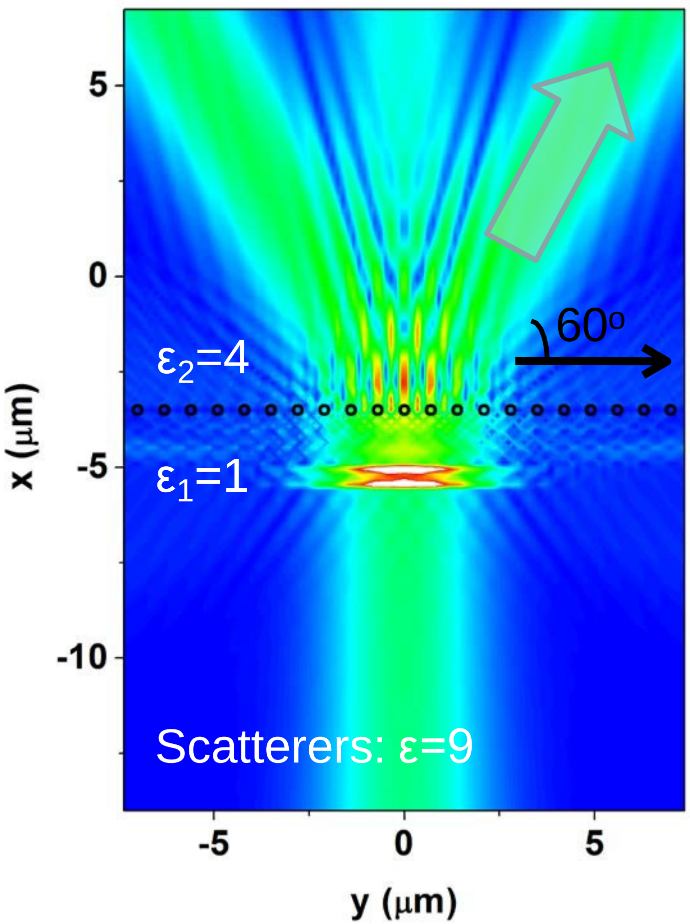
λ=400 nm
d=500 nm

$$\sin\theta = \frac{m\lambda}{d}$$



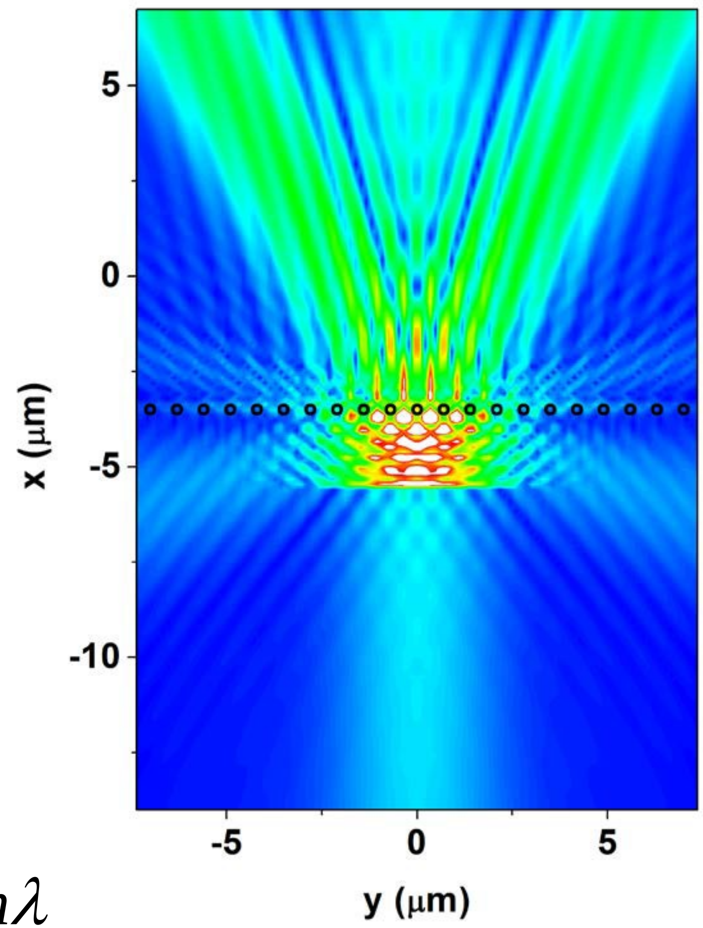
λ=700 nm
d=500 nm

Interaction of Light with Matter
Diffraction



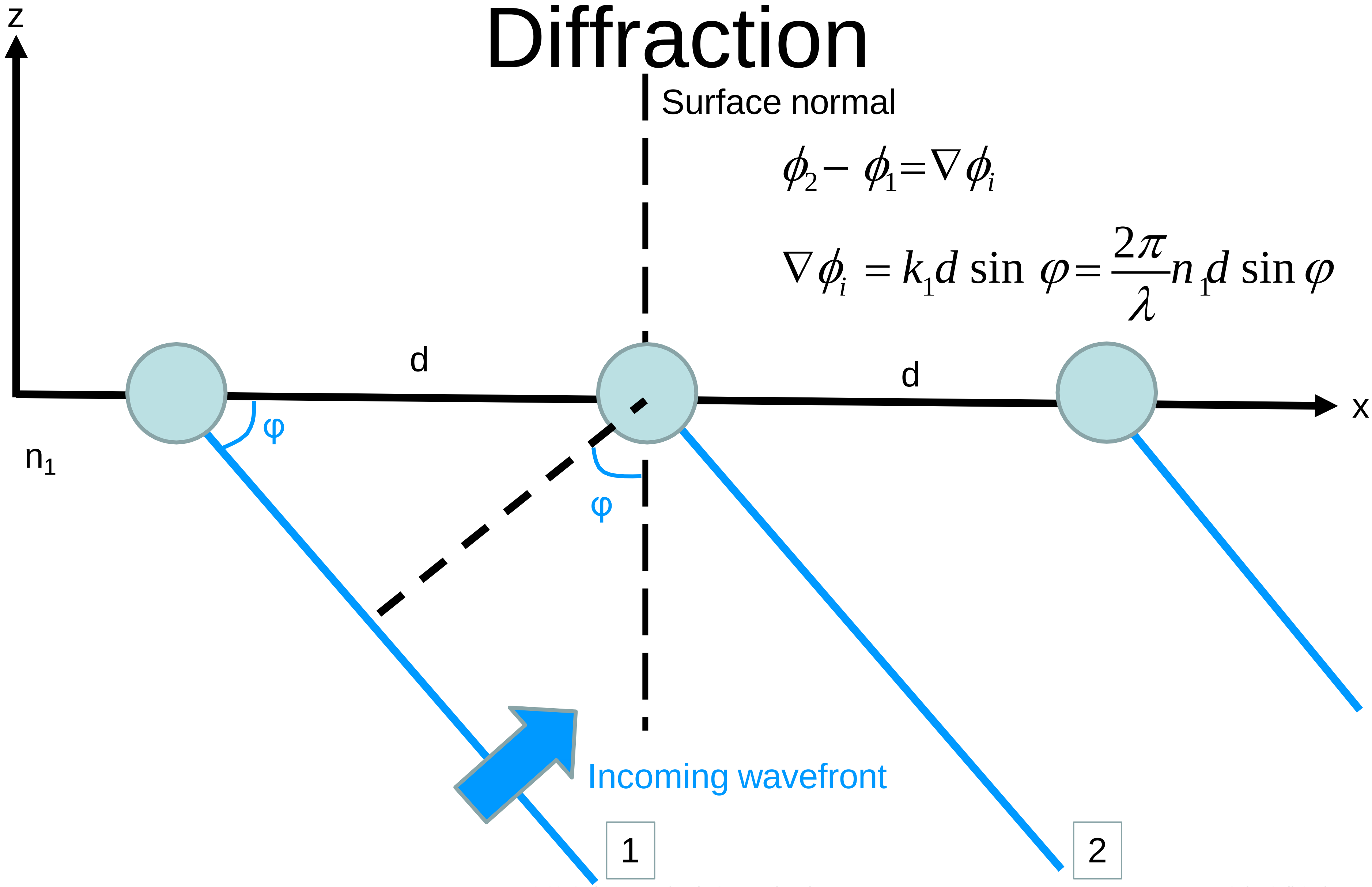
λ=400 nm
d=700 nm

$$\sin\theta = \frac{m\lambda}{n_2 d}$$



λ=700 nm
d=700 nm

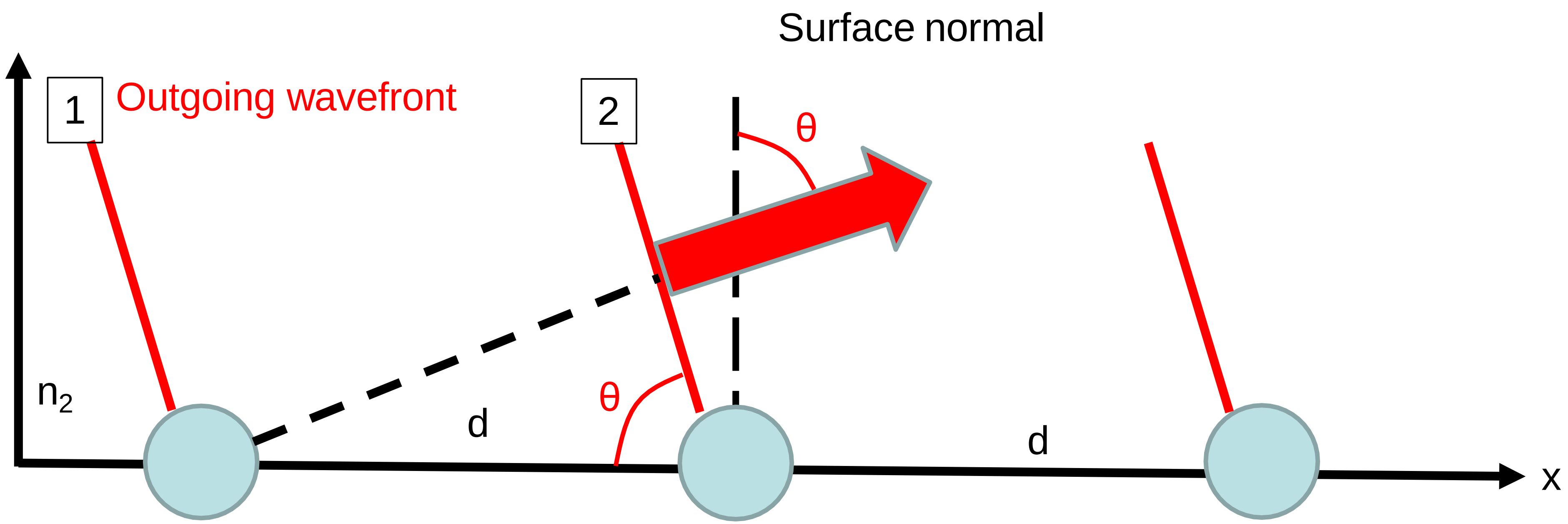
Interaction of Light with Matter
Diffraction



$$\phi_2 - \phi_1 = \nabla \phi_i$$

$$\nabla \phi_i = k_1 d \sin \varphi = \frac{2\pi}{\lambda} n_1 d \sin \varphi$$

Interaction of Light with Matter
Diffraction



$$\phi_2 - \phi_1 = \nabla \phi_i$$

$$\phi_1 - \phi_2 = \nabla \phi_o$$

$$\nabla\phi_i = k_1 d \sin\varphi = \frac{2\pi}{\lambda} n_1 d \sin\varphi$$

$$\nabla\phi_o = k_2 d \sin\theta = \frac{2\pi}{\lambda} n_2 d \sin\theta$$

Interaction of Light with Matter
Diffraction

$$\phi_1 - \phi_2 = \nabla\phi_o$$

$$\nabla\phi_o = k_2 d \sin\theta = \frac{2\pi}{\lambda} n_2 d \sin\theta$$

$$\phi_2 - \phi_1 = \nabla\phi_i$$

$$\nabla\phi_i = k_1 d \sin\varphi = \frac{2\pi}{\lambda} n_1 d \sin\varphi$$

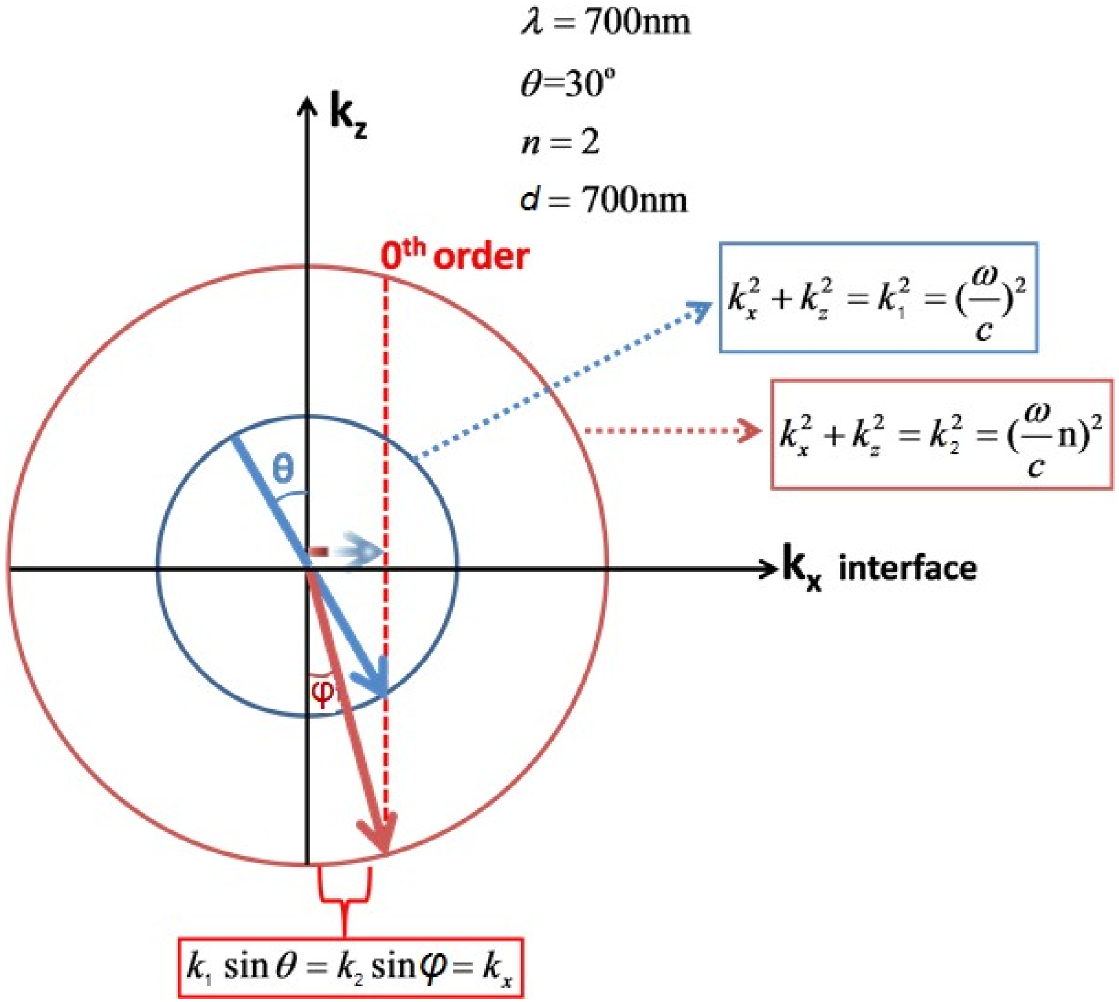
$$\nabla\phi_o - \nabla\phi_i = 2\pi m$$

$$k_2 d \sin\theta - k_1 d \sin\varphi = 2\pi m$$

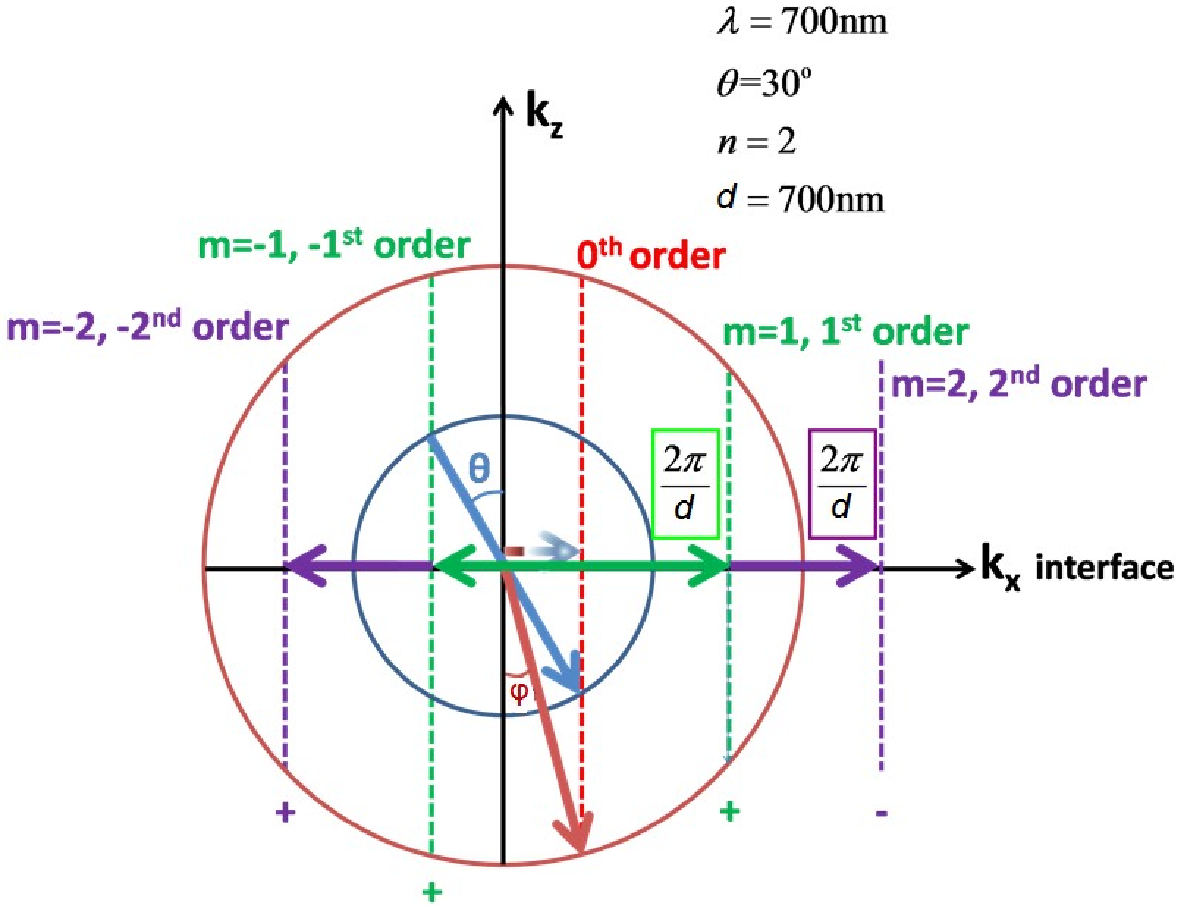
$$k_2 \sin\theta = k_1 \sin\varphi + \frac{2\pi m}{d}$$

Outgoing tangential | Incoming tangential | Phase term

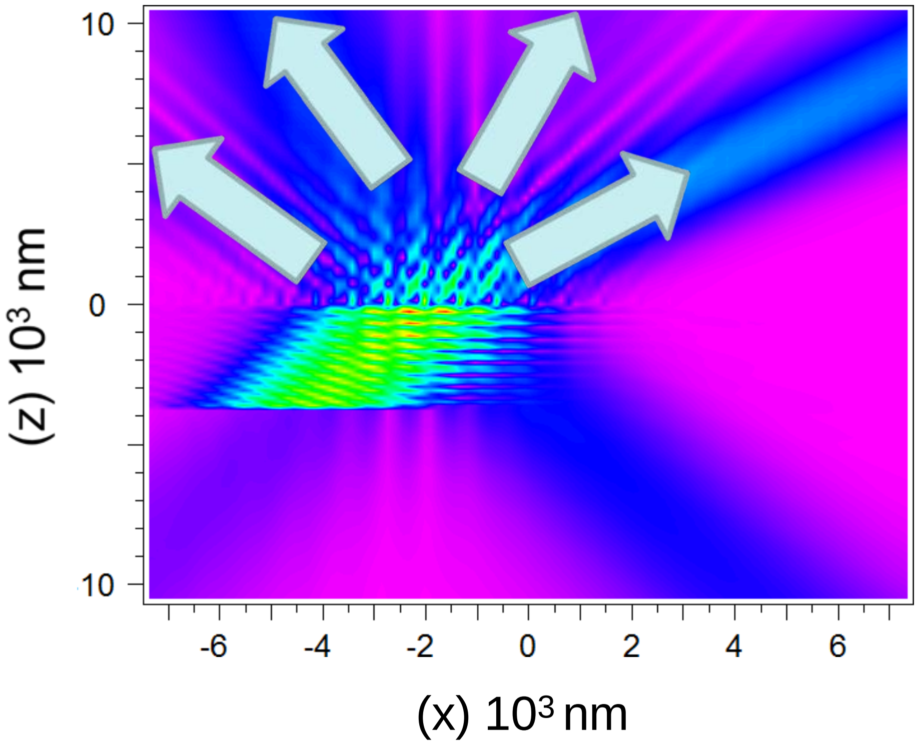
Interaction of Light with Matter
Diffraction



Interaction of Light with Matter
Diffraction



Interaction of Light with Matter
Diffraction



$\lambda = 700nm$

$\theta = 30^{o}$

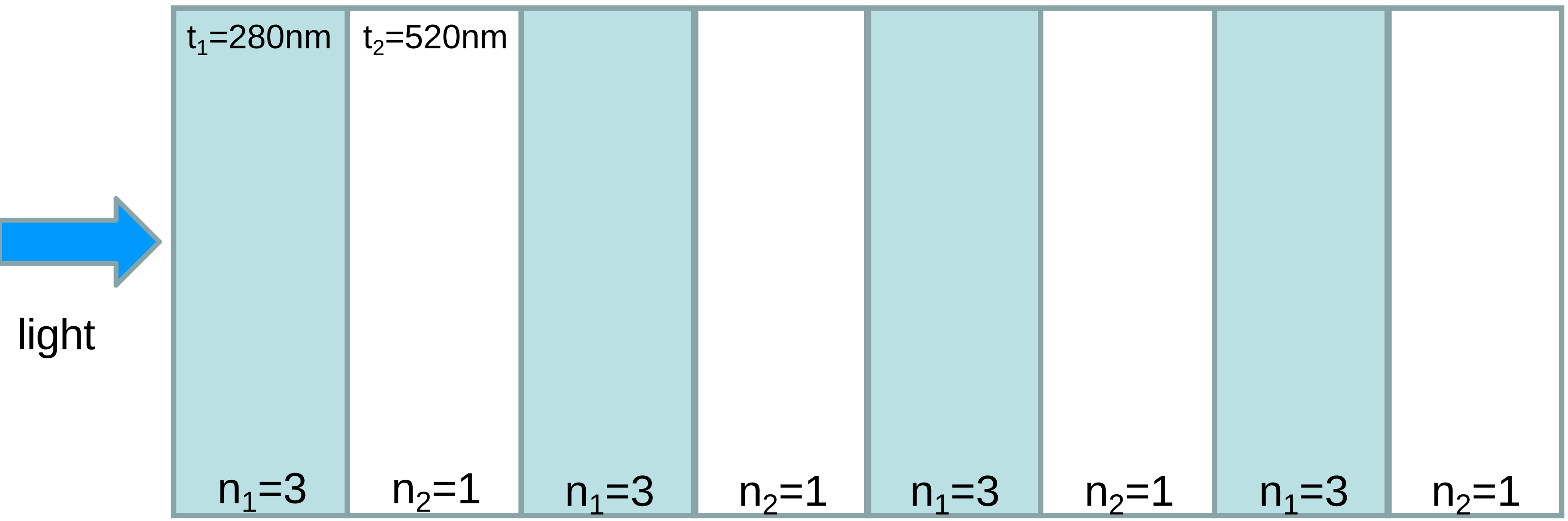
$d = 700nm$

$n_1 = 1$

$n_2 = 2$

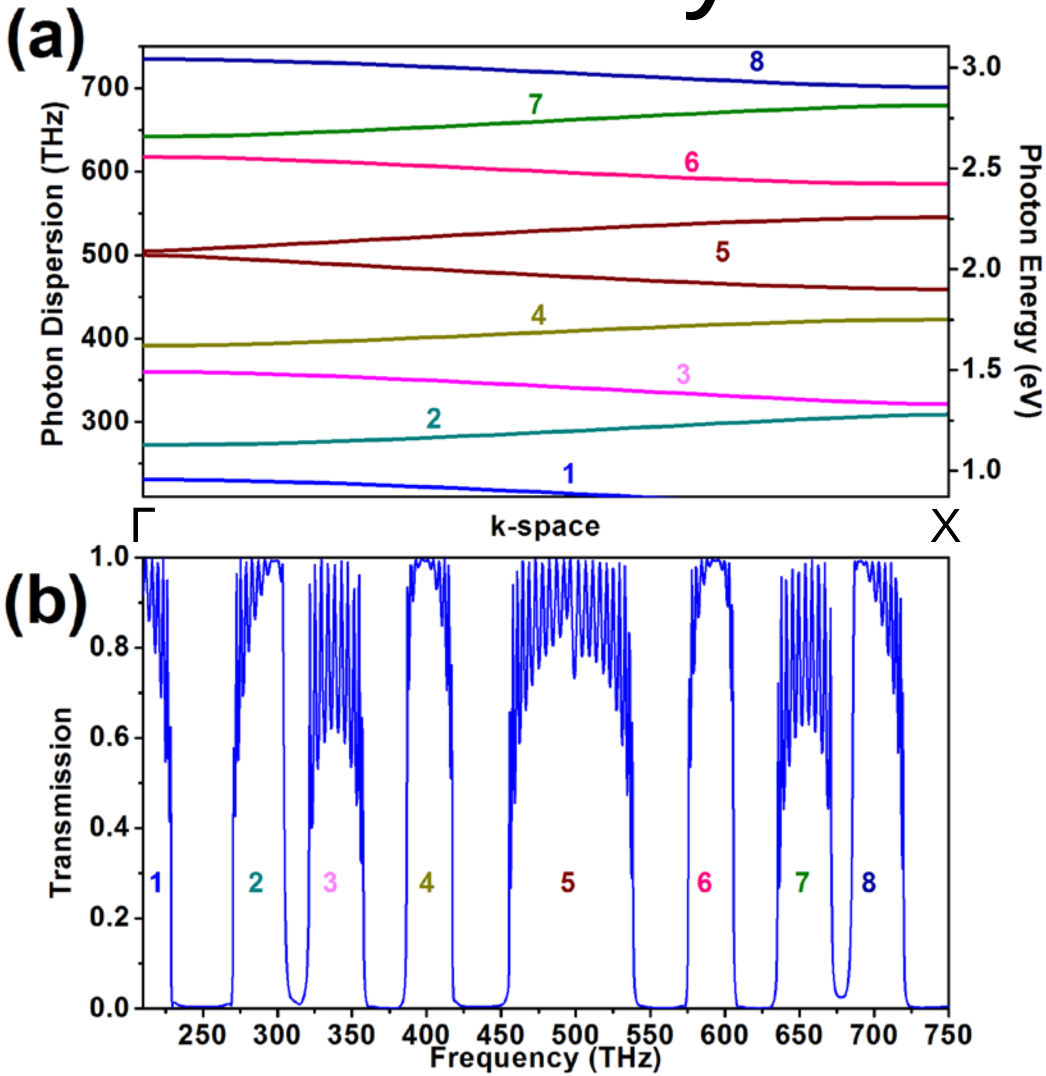
	$\sin^{-1}(\sin\theta/n_2 + m\lambda/n_2 d)$
0^{th} order	14 degrees
1^{st} order	(m=1) 48 degrees, (m=-1) -14 degrees
2^{nd} order	(m=2) NA, (m=-2) 48 degrees

Interaction of Light with Matter
Photonic Crystals

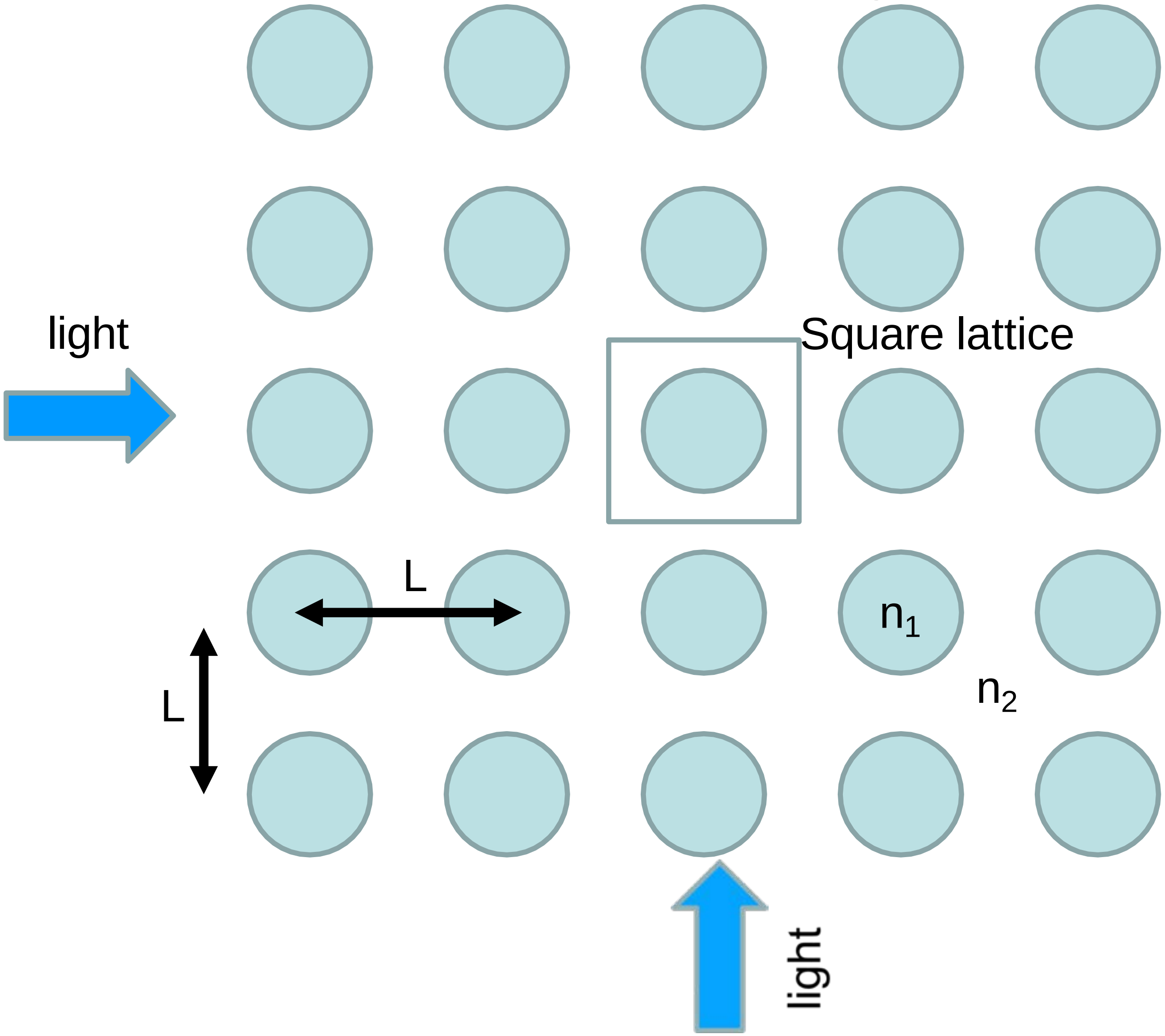


What happens if we stack layers along the propagation direction?

Interaction of Light with Matter
Photonic Crystals

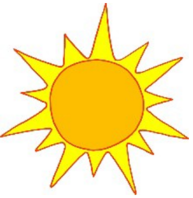


Interaction of Light with Matter
Photonic Crystals

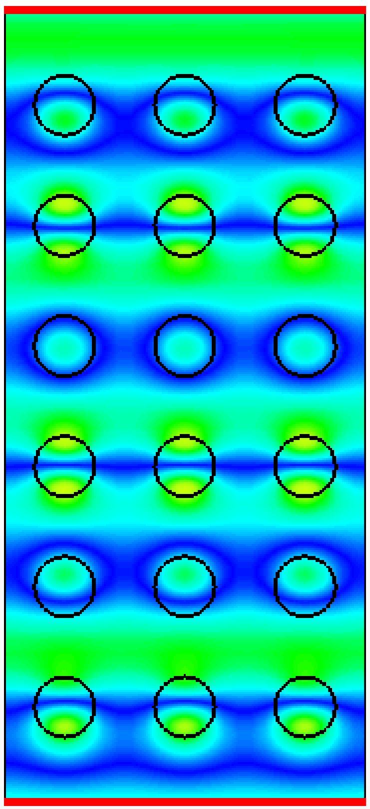


$$L = 1400nm$$
$$n_1 = 3$$
$$n_2 = 1$$

Interaction of Light with Matter Photonic Crystals



Periodic Dielectric Function



Bloch Functions/Waves

Bulk material

$$\vec{E}(\vec{r},t) = \mathrm{Re}\left\{E_0 e^{j\omega t} e^{-j\vec{k}\cdot\vec{r}}\right\}, k = \frac{2\pi}{\lambda} n$$

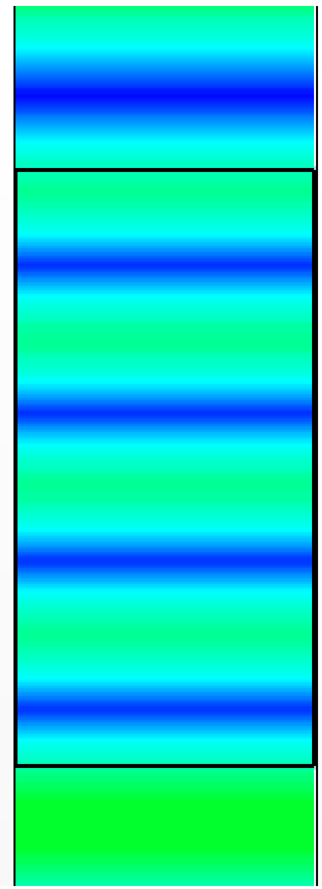
Periodic Structure

$$\varepsilon(\vec{r}) = \varepsilon(\vec{r} + L) \longrightarrow \vec{E}(\vec{r}) = \vec{E}(\vec{r} + L)$$

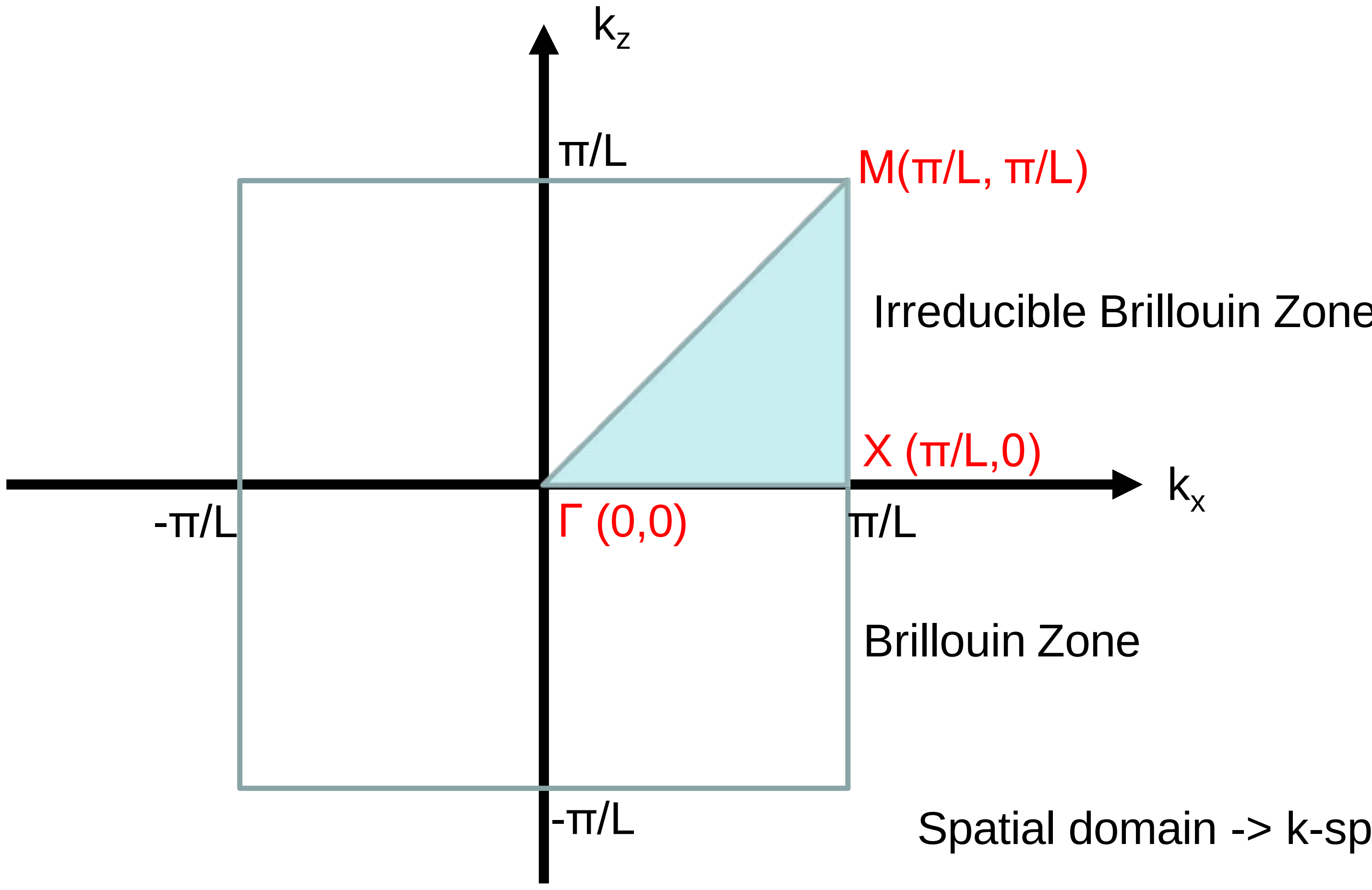
$$\vec{E}(\vec{r},t) = Re\left\{u(\vec{r}) e^{j\omega t} e^{-j\vec{k}\cdot\vec{r}}\right\}$$

$$u(\vec{r}) = u(\vec{r} + L), k = \frac{2\pi}{L}$$

Bulk Material

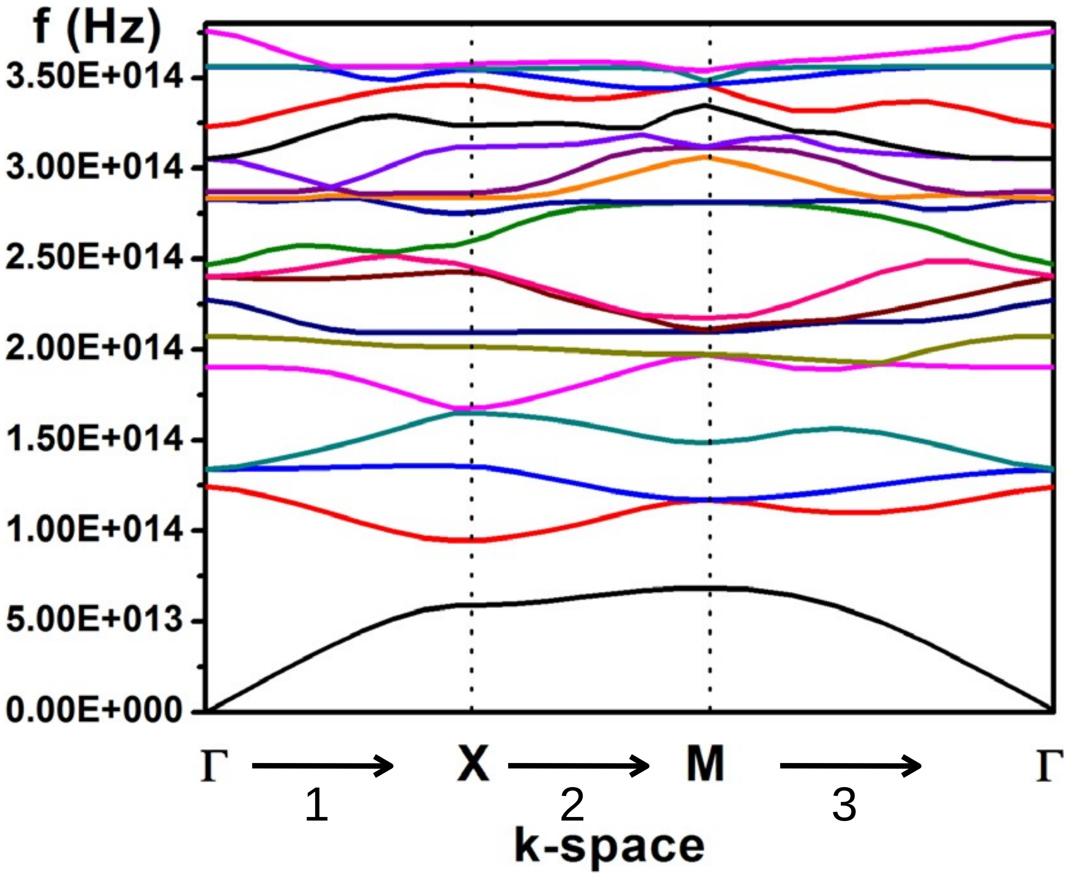


Interaction of Light with Matter
Photonic Crystals

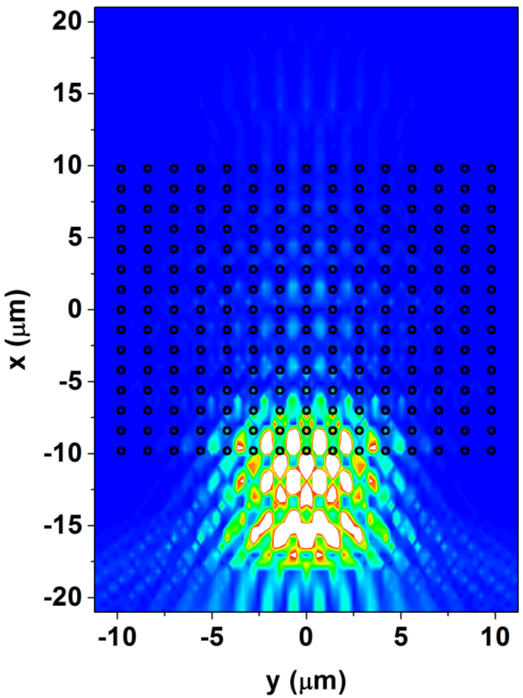


Spatial domain -> k-space

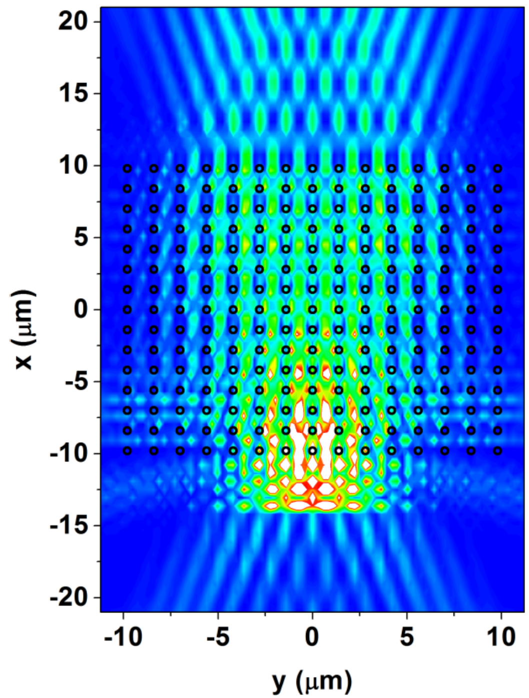
Interaction of Light with Matter
Photonic Crystals



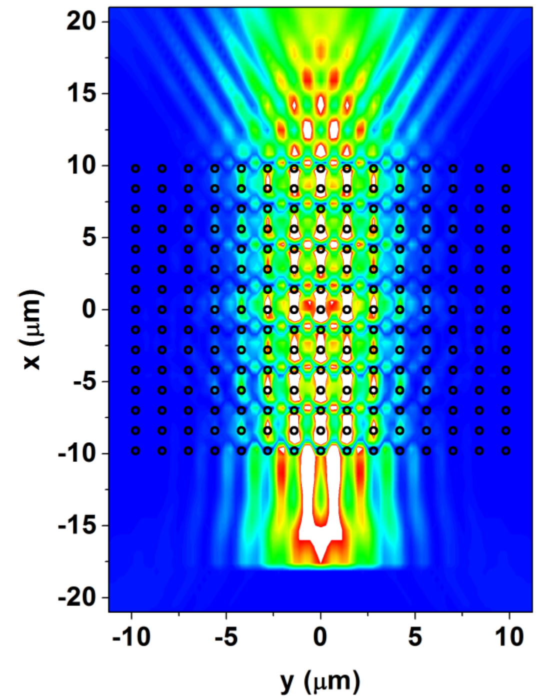
Interaction of Light with Matter
Photonic Crystals



λ=400 nm
L=1400 nm



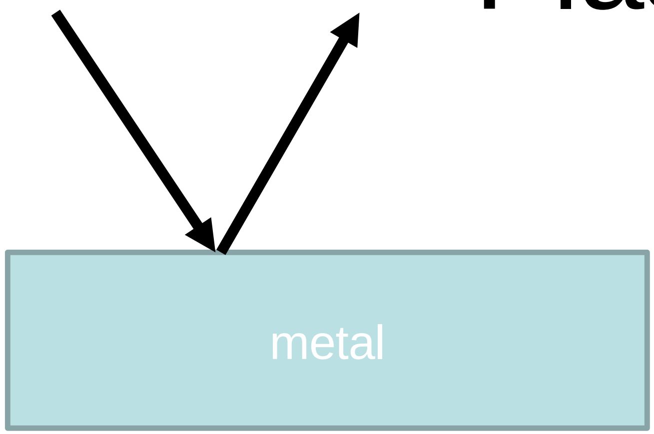
λ=700 nm
L=1400 nm



λ=1000 nm
L=1400 nm

Some inherent modes of the PC are coupled good enough to the impinging light, resulting in good transmission results.

Interaction of Light with Matter
Plasmonics



Normally:

- Metals totally reflect the incoming light
- High conductivity=> Electrons react to the incoming light, almost no penetration.

At smaller wavelengths:

-The electron cloud and the incoming light interact. They form SPPs.

-SPP is attached to the metal, can cover long distances.

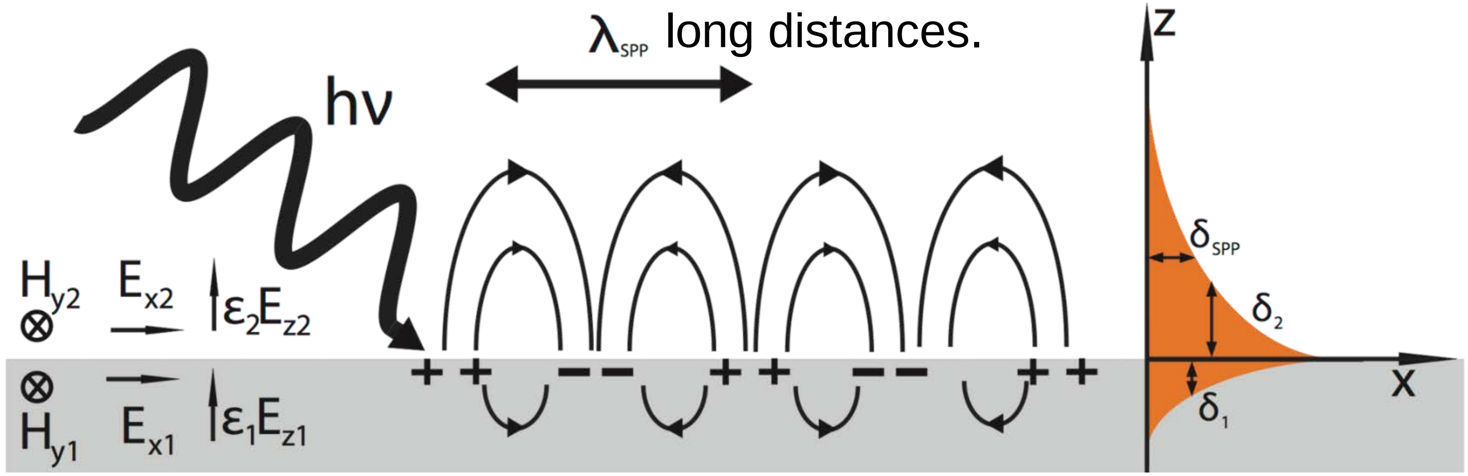


Fig. 23

- Fig. 23 taken from: http://en.wikipedia.org/wiki/Surface_plasmon#mediaviewer/File:Sketch_of_surface_plasmon.png

Interaction of Light with Matter
Plasmonics

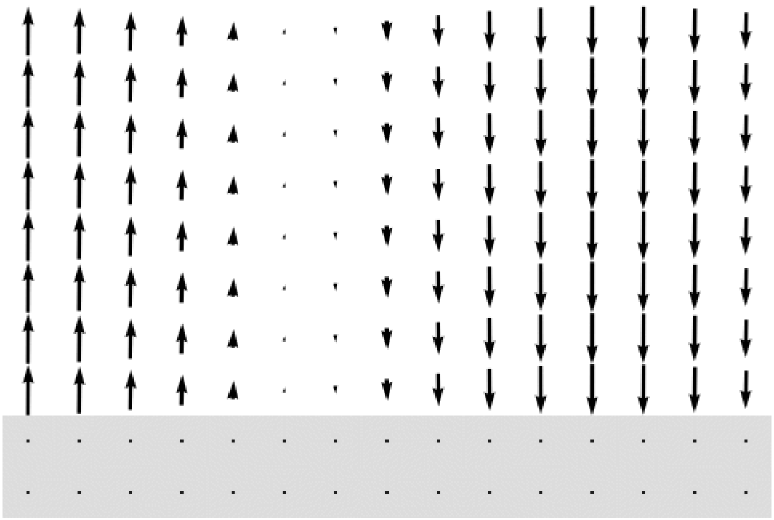


Fig. 24

Silver as PEC

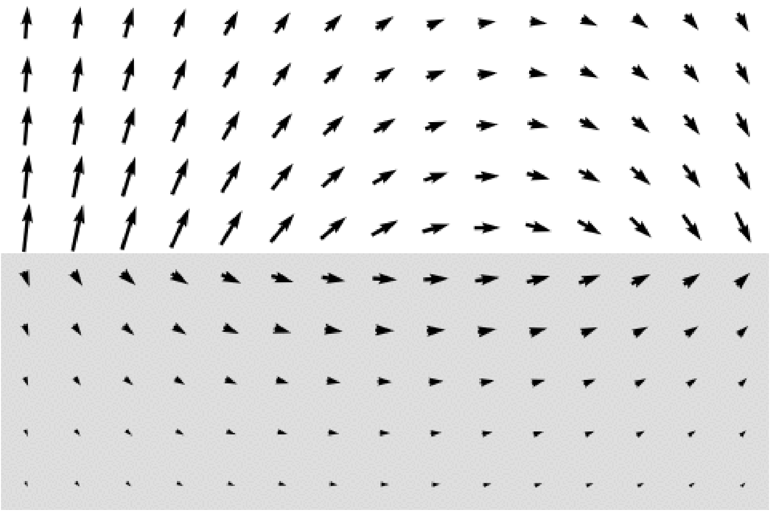
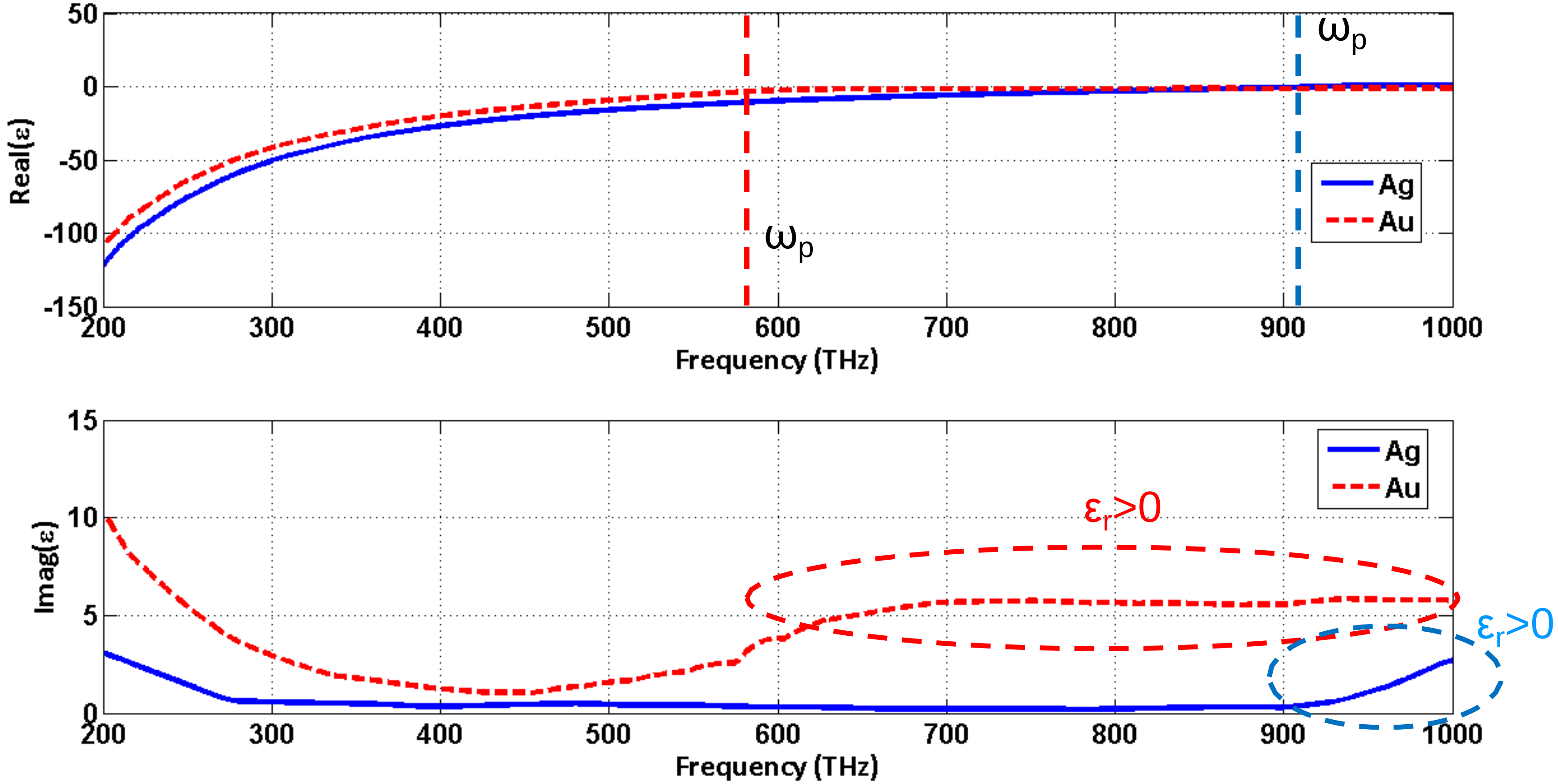


Fig. 25

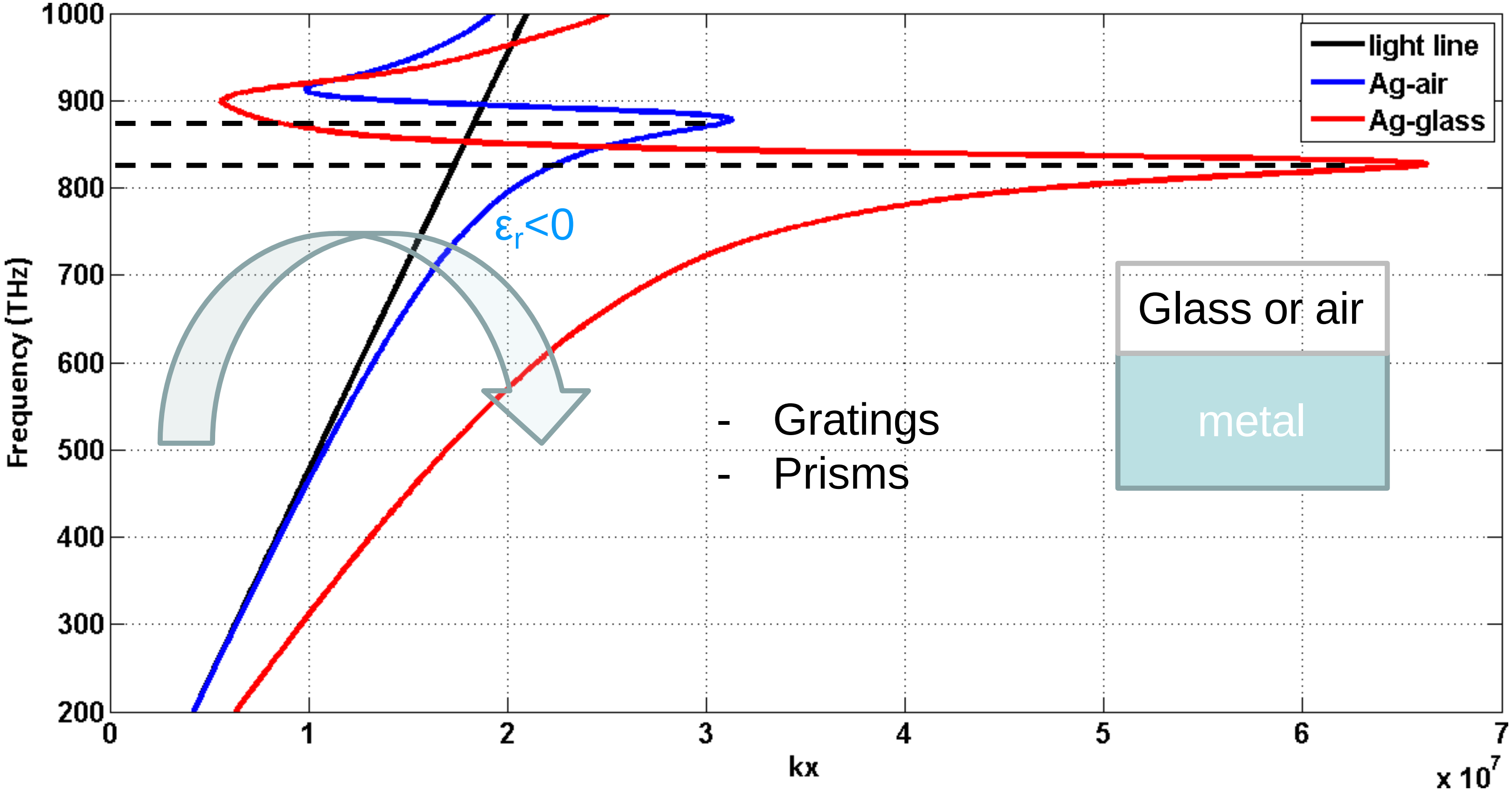
SPP on Silver

- Fig. 24 and 25 taken from: http://en.wikipedia.org/wiki/Surface_plasmon_polariton

Interaction of Light with Matter
Plasmonics



Interaction of Light with Matter
Plasmonics



- Gratings
- Prisms

Interaction of Light with Matter
Plasmonics

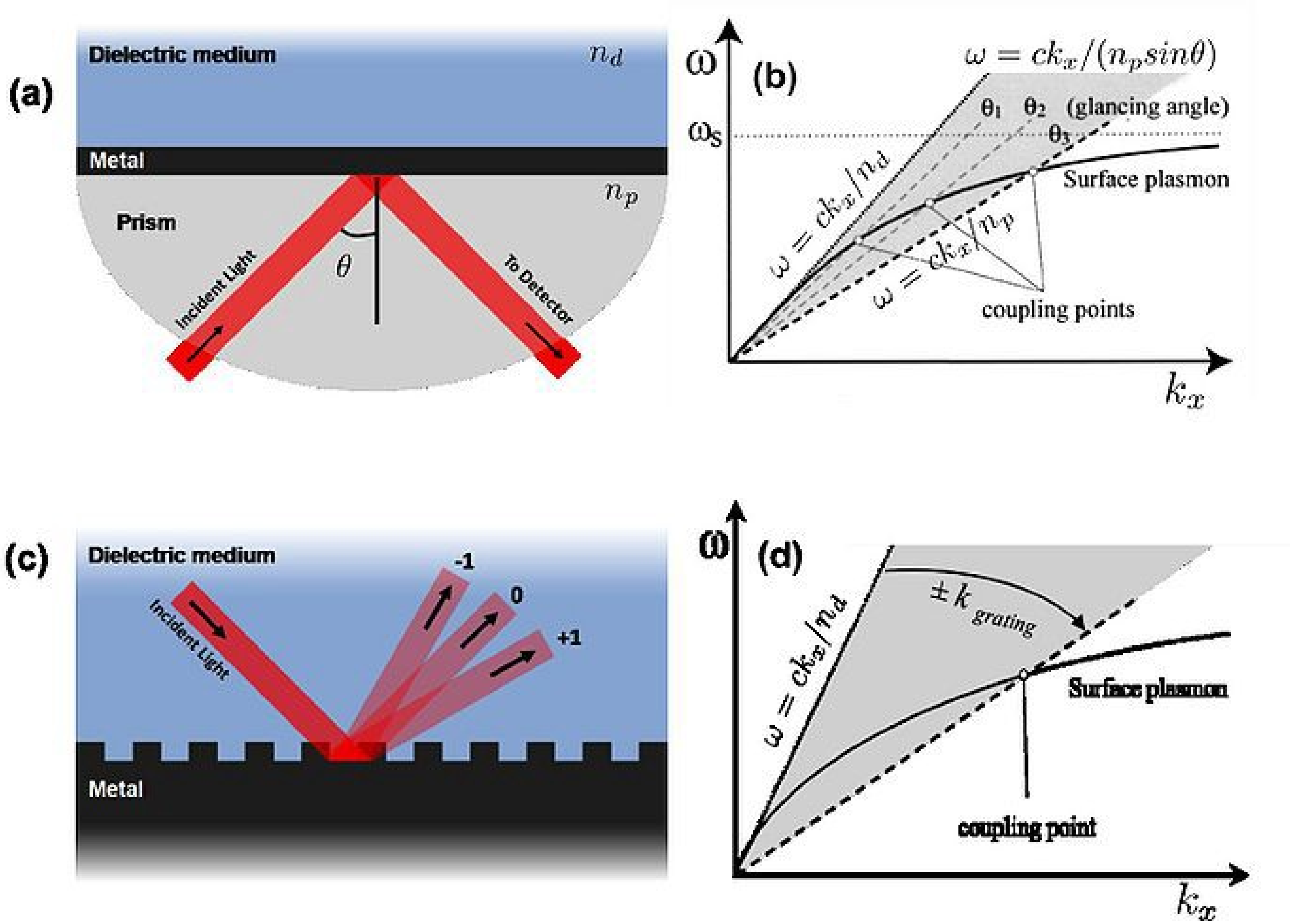
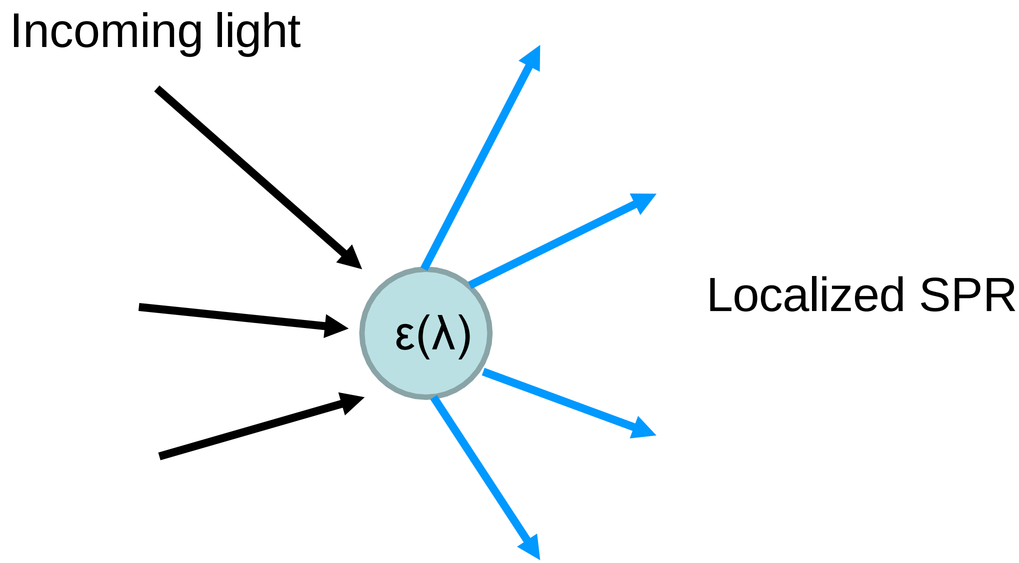


Fig. 26

- Fig. 26 taken from: https://www.thermalfluidscentral.org/encyclopedia/index.php/Surface_plasmon_(or_phonon)_polaritons

Interaction of Light with Matter
Plasmonics / Mie Scattering

Fig. 27

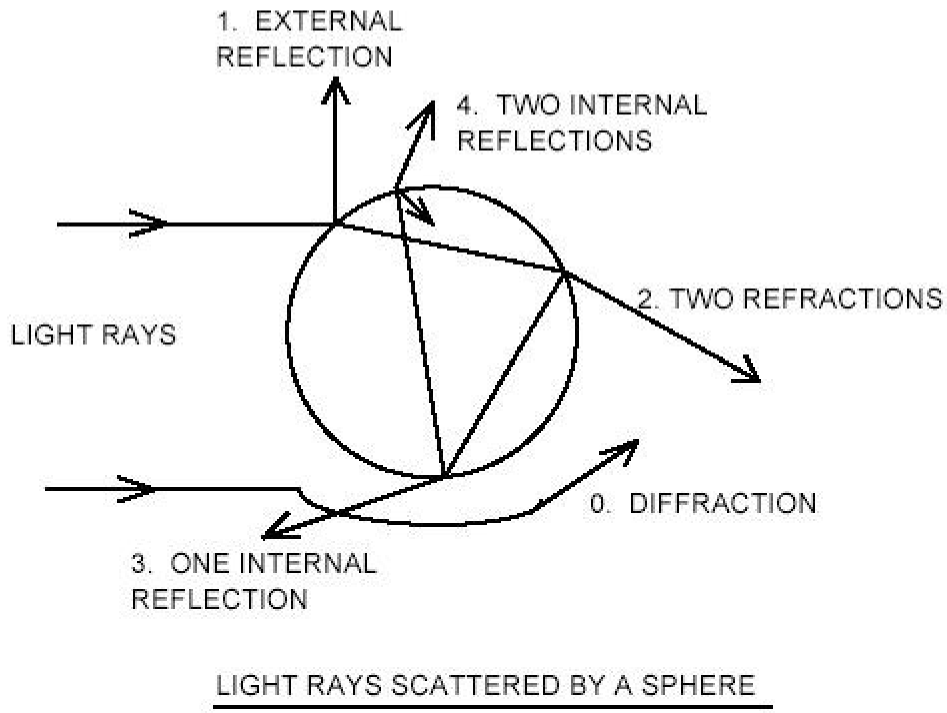


Scattering efficiency: $$Q(\lambda) = \frac{8}{3}\left(\frac{2\pi r}{\lambda}\right)^4 \left|\frac{\varepsilon(\lambda)-1}{\varepsilon(\lambda)+2}\right|^2$$

r:radius

- Fig. 27 taken from:
http://eniyavaikooral.blogspot.com/search/label/%E0%AE%AE%E0%AE%B0%E0%AF%81%E0%AE%A4%E0%AF%8D%E0%AE%A4%E0%AF%81%E0%AE%B5%E0%AE%AE%E0%AF%8D#axzz3FPFmWpJb

Interaction of Light with Matter
Mie Scattering



Dielectric spheres

Interaction of Light with Matter
Mie Scattering

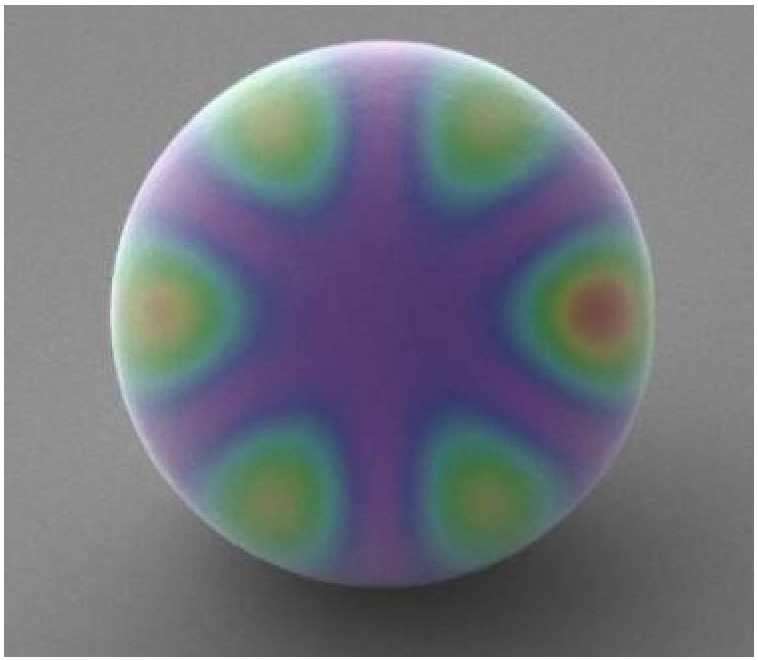


Fig. 28

Mie Resonances

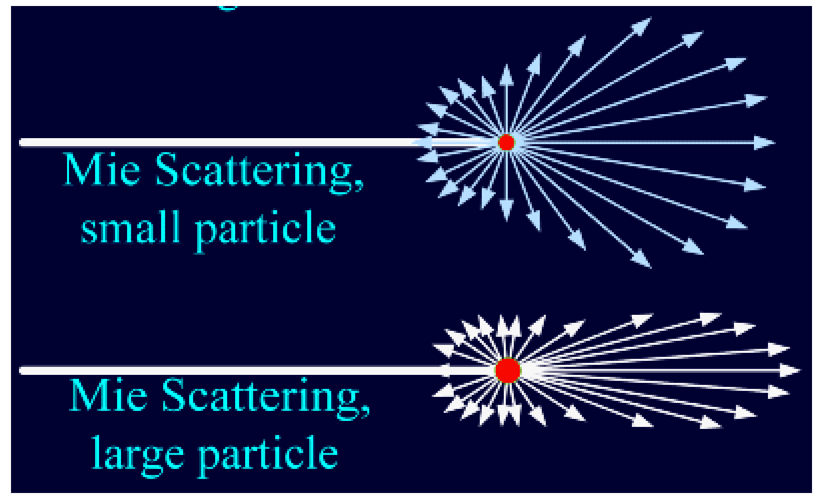


Fig. 29

$r \leq \lambda$

-Fig. 28 taken from: http://www.laserfocusworld.com/articles/2014/03/forming-silicon-into-microscopic-spheres-forces-it-to-absorb-infrared-light-enabling-better-photodetectors-and-solar-cells.html
-Fig. 29 original at: http://www.gamedev.net/topic/642193-what-causes-light-scattering-and-absorbtion/