Applications of Fourier Theory

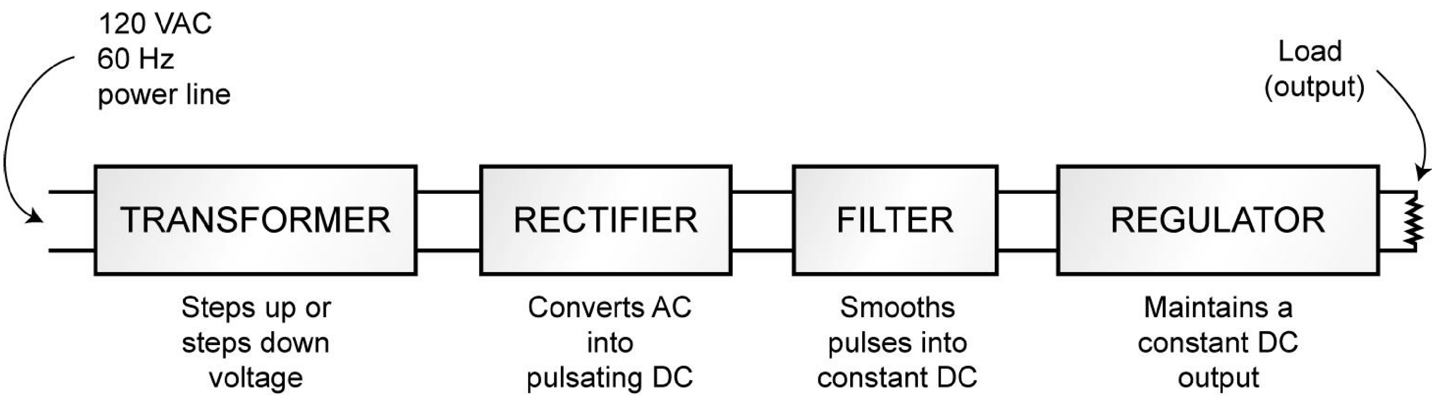
How Fourier Can Be Helpful

In explaining how electronic circuits work, most textbooks and instructor lectures rely mostly on time domain explanations. These discussions track voltage and current changes such as the charging and discharging of capacitors, the induction of voltages in inductors and transformers, and the variation of currents in semiconductors.

While these discussions are useful, they often do not tell the whole story and, in fact, obscure the real action. By including a frequency domain discussion along with the time domain explanation, the whole truth becomes evident.

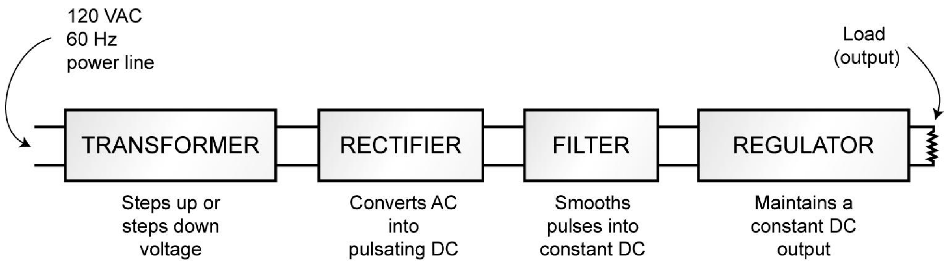
This section describes electronic applications along with relevant frequency domain and Fourier explanations.

Power Supply Filters



A discussion of the graphic is presented in the pages that follow. You can print this graphic for study purposes before going on.

Power Supply Filters



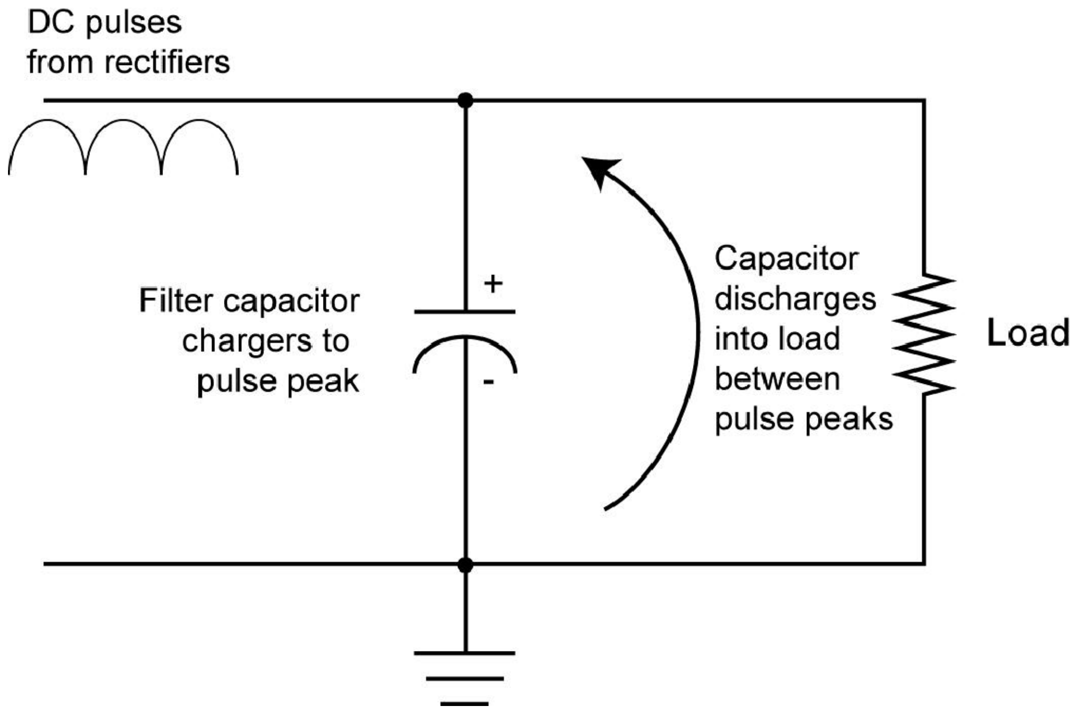
A power supply is a circuit that converts the AC power line voltage into one or more DC voltages used to operate electronic circuits. All electronic equipment contains some kind of power supply.

A transformer either steps up or steps down the voltage. Diodes are used in a rectifier to change the AC into DC. As you saw earlier, rectification converts the AC sine wave into DC pulses.

The next step is to filter the pulses into a constant DC voltage level. The filtering is usually carried out by a large capacitor.

A regulator at the output is used to maintain a constant DC output in spite of any load or line input changes.

A Time Domain Explanation of the Filter



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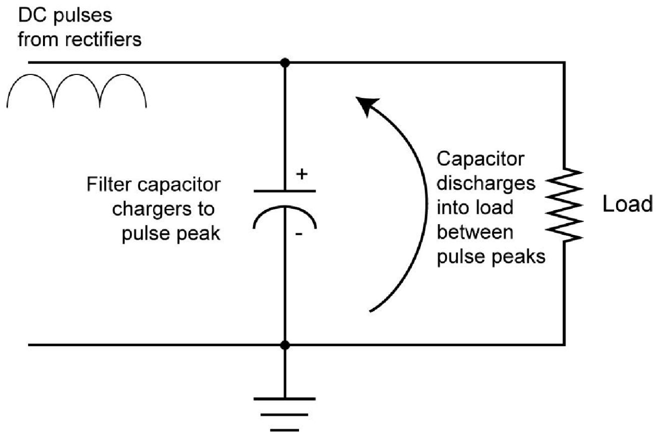
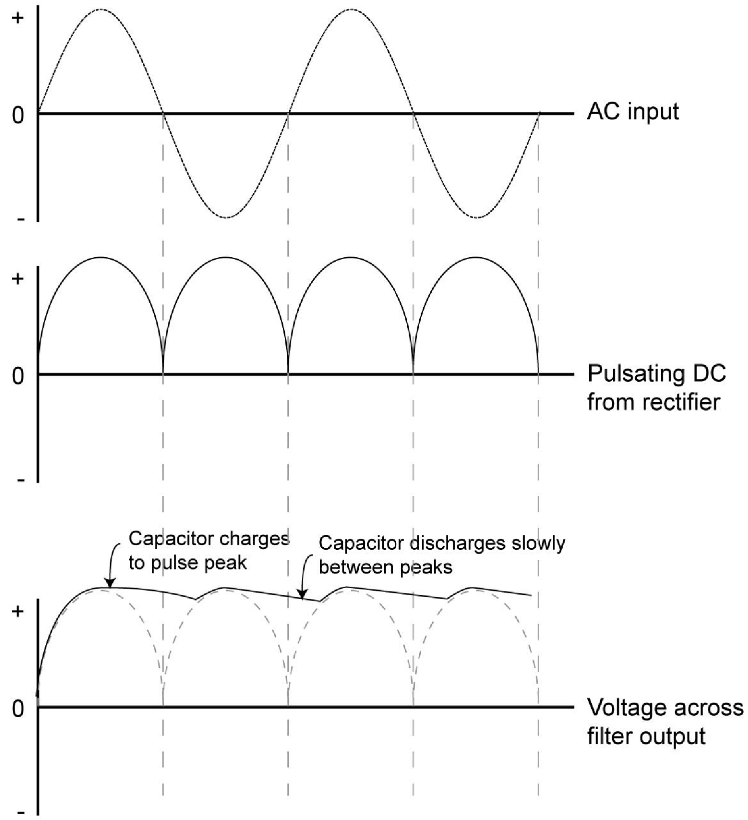
A Time Domain Explanation of the Filter

Most discussions of the filter operation focus on how the pulses from the rectifier charge the capacitor to the peak voltage.

The capacitor discharges into the load between the pulse peaks to help keep the voltage from changing. This is done by using a very large electrolytic capacitor that produces a very long time constant with the load. Between pulses, the capacitor discharges to help smooth the pulses into a more constant voltage.



A Time Domain Explanation (cont.)



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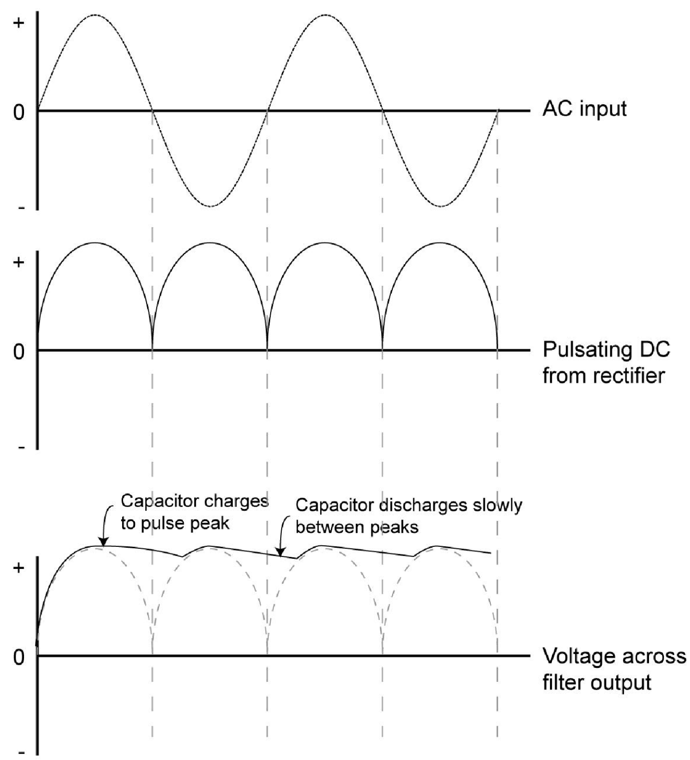
A Time Domain Explanation (cont.)

This figure shows the output pulses from a full wave rectifier and how the capacitor discharges when the pulses drop to zero.

The resulting DC output is very nearly a constant DC average value. There is some minor variation as the capacitor charges and discharges. This variation is known as ripple.

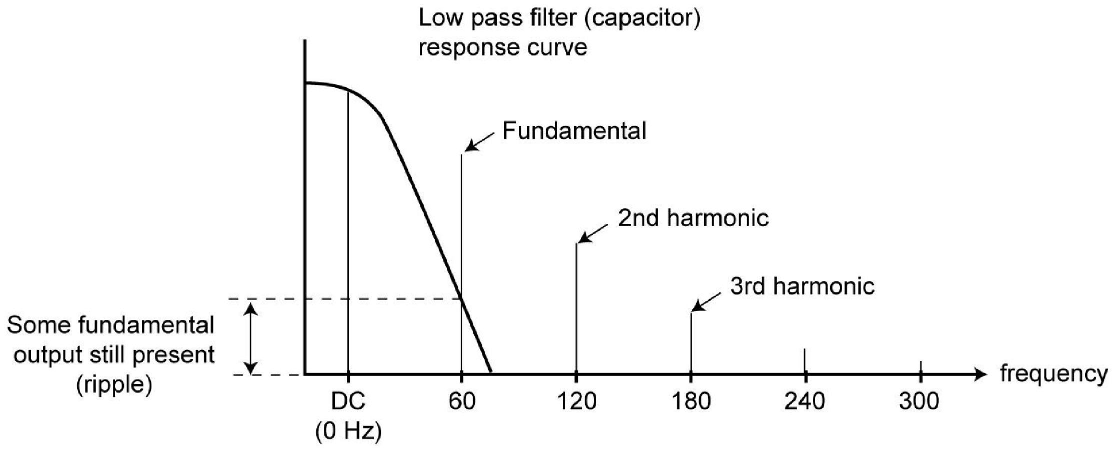


A Frequency Domain Explanation of the Filter

Another way to view the capacitor is as a low pass filter. The capacitor value is chosen to set the cut-off frequency to some value below the fundamental frequency of the rectifier pulses.

The goal is to eliminate the pulses and retain only a constant DC. Recall from the discussion of the rectified sine waves earlier that the Fourier expression has a DC component. This DC component is the desired output. The fundamental as well as the harmonics are to be filtered out.

Frequency Response Curve



This figure shows the frequency response curve of the capacitor as a low pass filter superimposed on the frequency domain plot of the full wave rectifier spectrum.

Note that the DC component is passed completely but the fundamental and harmonics are greatly attenuated or removed.

The filter does pass a small amount of the fundamental (the ripple) that appears across the load.

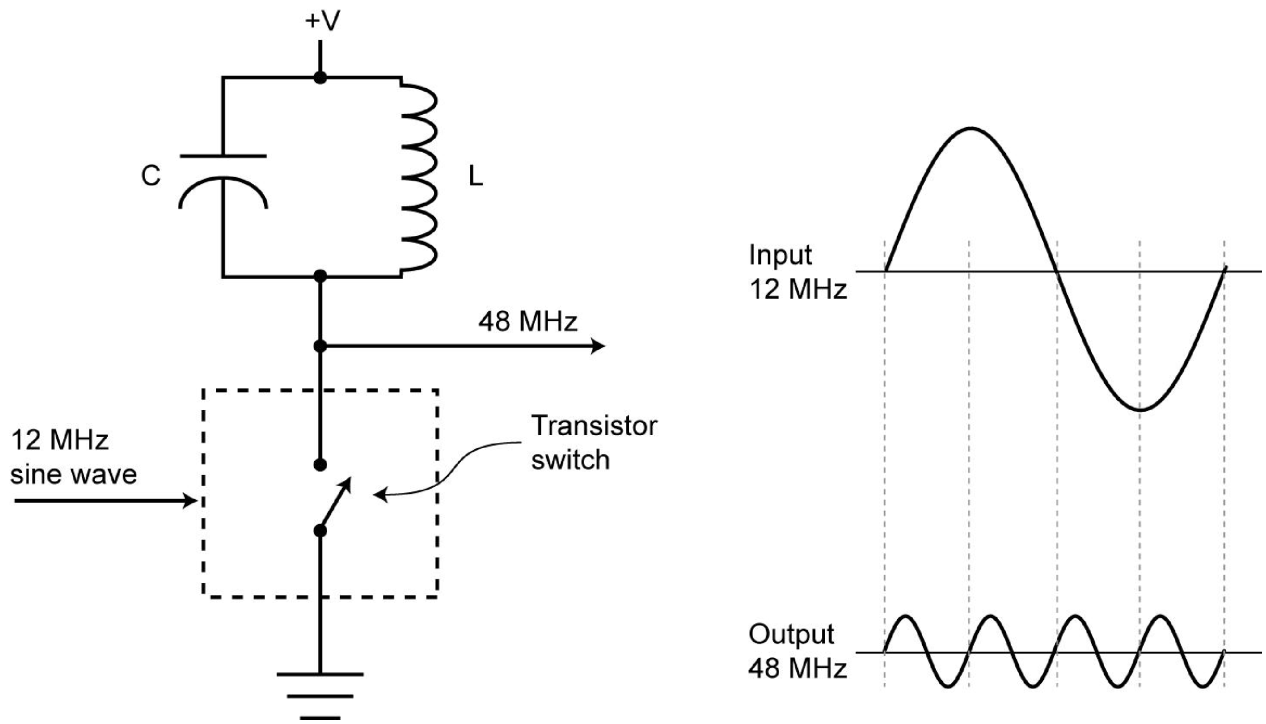
A Frequency Multiplier

A frequency multiplier is a circuit that takes a fundamental frequency as the input but produces an output as some integer multiple of the input frequency.

For example, a frequency doubler produces an output frequency twice the input. A frequency tripler produces an output frequency at three times the input frequency.

Frequency multipliers are widely used in radio transmitters to achieve higher output frequencies from lower frequency crystal oscillators.

A Frequency Multiplier



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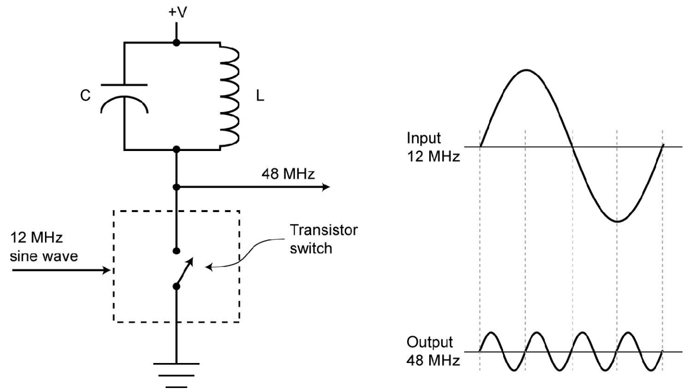
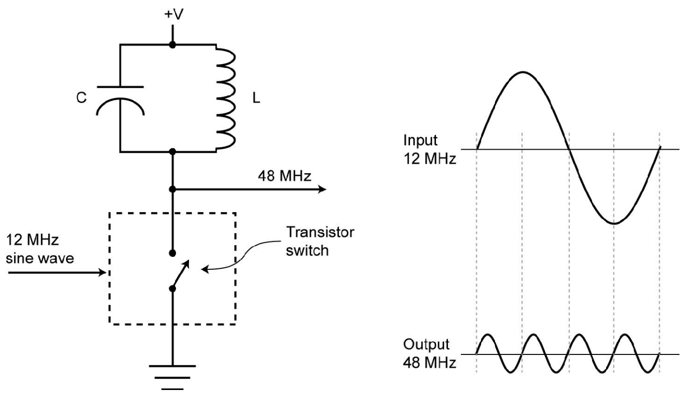
A Frequency Multiplier



A common way to produce a frequency multiplier is to use a switch to pulse a resonant circuit at the desired higher frequency.

This figure shows a parallel tuned circuit resonant to the desired output frequency of 48 MHz. Since the input is a 12 MHz sine wave, this circuit is a quadrupler.

Time Domain Explanation of the Multiplier

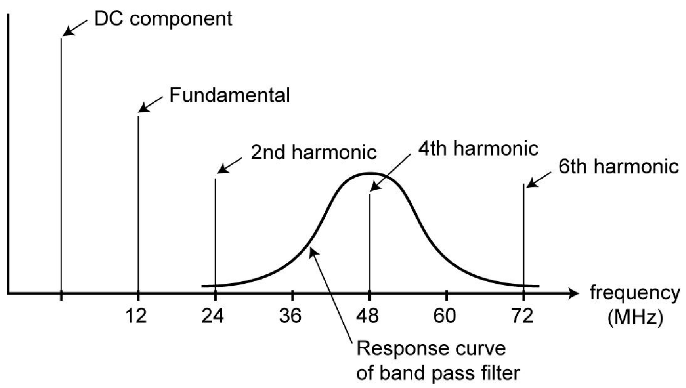


The 12 MHz sine wave input drives a transistor switch. When the switch is turned on, the DC voltage +V is applied to the tuned circuit. The capacitor is charged to +V.

When the switch opens, the tuned circuit "rings" or oscillates at its resonant frequency (48 MHz). The capacitor discharges into the inductor producing a magnetic field. The magnetic field collapses and recharges the capacitor in the opposite direction.

This exchange of energy between the capacitor and inductor produces a sine wave output at the resonant frequency.

Frequency Domain Explanation of the Multiplier



Another view of the operation of the multiplier shows the transistor switch effectively distorts the sine wave into pulses. For example, the transistor switch may turn on during the positive part of the input sine wave and be off during the negative part. The current flowing closely resembles half wave pulses such as those shown for the half wave rectifier. These pulses are rich in even harmonics meaning that the output contains the fundamental, the 2nd harmonic 24 MHz, the 4th harmonic, the 6th harmonic and so on. The parallel resonant circuit acts as a selective band pass filter which is tuned to 48 MHz. Therefore, the filter passes only the 48 MHz harmonic and attenuates the fundamental and all other harmonics.

Transmitting Binary Pulses Over a Cable

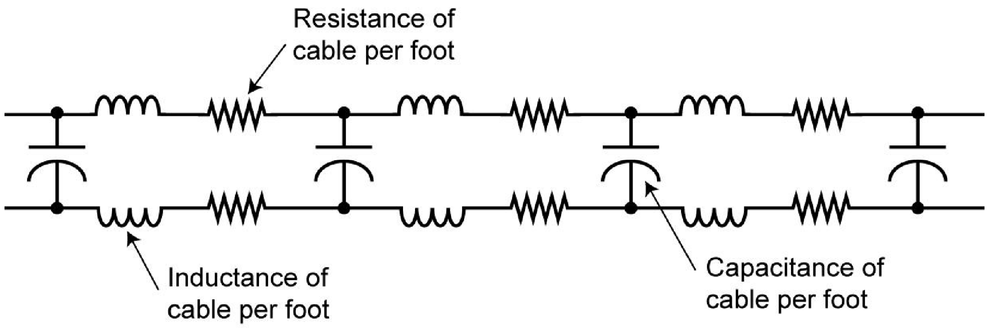
A common application is the transmission of digital, mainly binary pulses, over a cable. This is typical in computers, networks, and other data communications system such as digital telephony. Such binary pulse trains produce a very wide spectrum of harmonics.

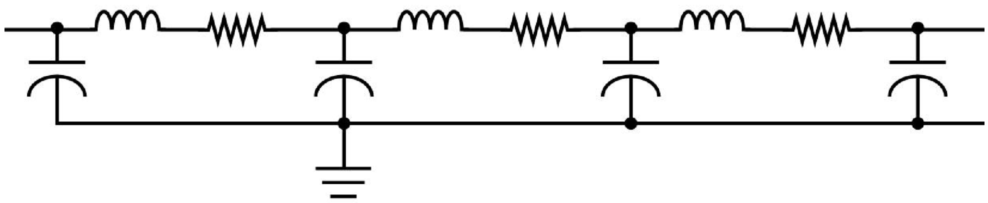
However, the twisted pair cable commonly used in computer networks and the coax cable commonly used in networks and many telephone systems actually have a finite bandwidth.

Most cables act as low pass filters and, as a result, naturally attenuate the higher harmonics. As a result, the cable distorts the binary signal applied to its input.

To preserve the characteristics of the output pulse, the cable must have sufficient bandwidth to avoid filtering out too many of the higher harmonics.

Cable Equivalent Circuit and Characteristics





Simplified equivalent circuit : a low pass filter

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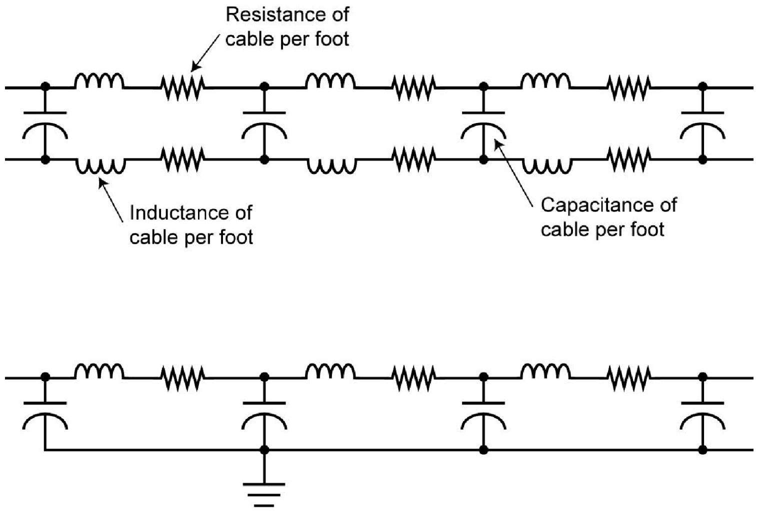
Cable Equivalent Circuit and Characteristics



Simplified equivalent circuit : a low pass filter

This figure shows the equivalent circuit of a two wire cable such as a twisted pair or coax cable. The capacitors in the diagram represent the distributed capacitance between the two wires. A capacitor is any two conductors separated by an insulator. This condition exists in any two wire cable.

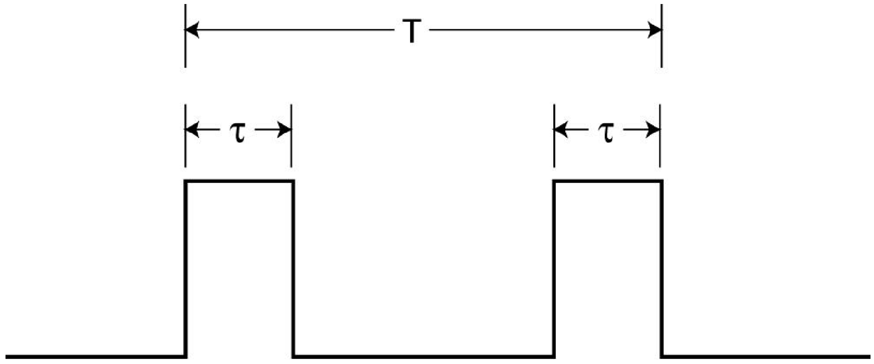
The Cable as a Low Pass Filter

The wires also exhibit inductance. Although not coiled as in most inductors, the wires nevertheless do have self inductance.

The wires will also have some resistance. The resulting equivalent circuit is a low pass filter. All low pass filters have an upper cut-off frequency.

In the case of a cable, this cut-off frequency is dependent upon the characteristics of the cable (e.g. capacitance and inductance per foot) and especially the length of the cable. The longer the cable, the lower the cut-off frequency and the greater the attenuation of the higher harmonics.

Binary Signal Frequency and Bit Rate



T = period

τ = pulse width

Bitrate (bps) = $^1/_\tau$

The amount of distortion and attenuation of a cable is also dependent upon the frequency or bit rate of the binary signal to be transmitted over the cable.

The digital data to be transmitted over the cable is binary information. A voltage pulse represents a binary 1 while the absence of a voltage (or lower voltage) represents a binary 0.

The figure above shows a typical binary data signal to be transmitted.

Relating Bit Rate to Bandwidth

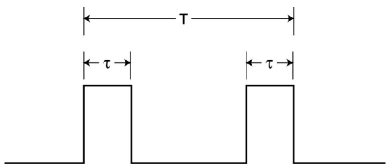
A key factor in binary data transmission is how many bits can be transmitted per second (faster = better). This is expressed as bits per second (bps) or the bit rate. The bit rate is related to the frequency of the pulses.

The binary pulse pattern changes continuously with the varying nature of the data. As a result, the frequency also changes. The bit rate is used to determine what the bandwidth of the cable should be to pass the data reliably.

The bit rate in bits per second (bps) is the reciprocal of the pulse width of the signal. This is τ in the figure. For example, if the pulse width is 50 nS, the bit rate is:

$$\text{bps} = 1/\tau = 1/50 \times 10^{-9} = 20{,}000{,}000 \text{ bps or } 20 \text{ Mbps}$$

The cable must have the bandwidth to pass a signal of 20 Mbps.



T = period
τ = pulse width
Bitrate (bps) = $1/\tau$

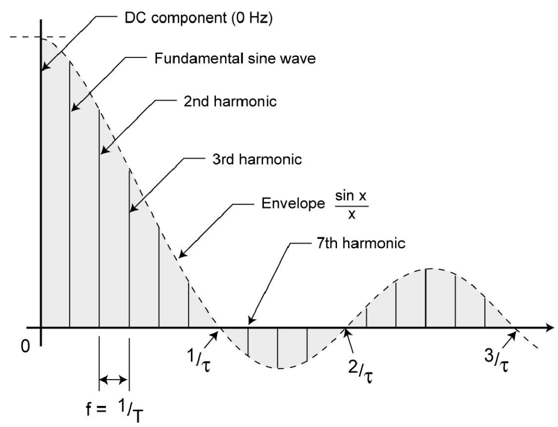
Determining Cable Bandwidth

In the spectrum of a rectangular pulse shown earlier, most of the energy (highest level harmonics) occur inside the first segment of the envelope between zero (DC) and $1/\tau$. As a general rule of thumb, the bandwidth of the cable should be at least this value. This can be stated:

$$BW = 1/\tau$$

In order to pass the signal with minimum distortion and maintain the rise and fall times, the cable should have a minimum cut-off frequency of $1/\tau$. For the example given in the previous frame, the cut-off frequency should be at least 20 MHz.

You can easily determine the type of cable and the length that will give you the desired results by using the cable manufacturer's data.



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