Frequency Domain Waveforms

Lab Summary: The purpose of this laboratory experience is to introduce the Fourier mathematical equations for non-sinusoidal waveforms in the frequency domain view. Calculated peak values of the first nine harmonics of a square wave will be determined. A circuit will be built and voltages will be measured using an oscilloscope and a function generator. Directions for completing the lab are provided. The data collected will be compared to the calculated values and used to answer pertinent questions.

Lab Goal: Construct a circuit and determine the differences between the calculated and measured output voltages of a square wave in the frequency domain view.

Learning Objectives

1. Use the Fourier mathematical expressions to calculate harmonic frequencies when given the voltage, frequency, and time.
2. Build a circuit with a ceramic filter.
3. Use a function generator to produce and fine-tune the output voltages at different harmonics.
4. Measure the peak values at different harmonics and record the values.
5. Discuss causes between differences in the calculated values and the measured values.

Grading Criteria

Your lab grade will be determined by your performance on the mathematical equations, the experiment, lab questions, and your written conclusion of the experiment.

Approximate Time Required: 3 hours

Equipment and Supplies

Equipment:
Dual-trace oscilloscope
Function generator
Frequency counter
Spectrum analyzer (if available)

Components:
Ceramic filter: TOKO CFMR
Resistors: 2 (values determined from data sheet)

Lab Preparation

1. Assemble all equipment and components.
2. Read Introduction (below)
3. Review Lab Procedures (below)

Introduction

The Fourier theory is an infinite mathematical series that Fourier used in his analysis and design of heat and thermodynamic systems. It was later discovered that it could be used in almost any engineering analysis including mechanical and electrical/electronic systems. The Fourier theory shows the relationship between the time and frequency domains. It provides a way to represent complex non-sinusoidal signals as the sum of harmonically related sine and/or cosine waves. In electronics, the Fourier theory can be used to explain how signals are filtered and processed. The Fourier series can be used to show how sine waves are used to make up the far more complex waves, such as the square, triangle, pulse, sawtooth, and other waves.

The Fourier theory states:

> Any complex repetitive, non-sinusoidal waveform may be expressed mathematically as a fundamental sine wave at the signal frequency plus an infinite number of harmonic signals of varying amplitudes and phases.

A general expression of the Fourier theory is:

$$y = A_o + \sum [A_n \sin 2\pi(nf)t + B_n \cos 2\pi(nf)t]$$

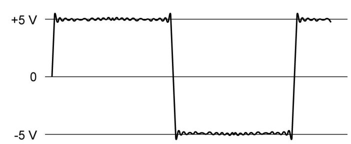
The term y is some signal that is a function of time or f(t). A_o is the DC component of the wave if it has one. The summation sign ($\sum$) indicates that a whole series of sine and cosine terms are to be added together. A is the sine amplitude, B is the cosine amplitude. The first term where n = 1 is the fundamental, the second term where n = 2 is the 2nd harmonic, etc. The term n designates the higher integer values of the harmonics.

The non-sinusoidal waveform most used to illustrate the Fourier theory is the square wave. If put through a rigorous Fourier analysis, the square wave is shown to be made up of a fundamental sine wave and an infinite number of odd harmonics (3rd, 5th, etc.).

The Fourier expression for a square wave is:

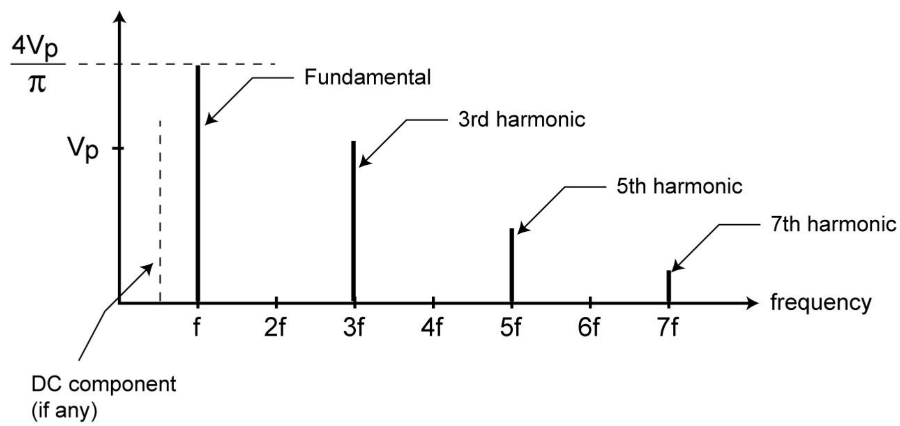
$$y = V_p + 4V_p/\pi\ [\sin\omega t + (\sin 3\omega t)/3 + (\sin 5\omega t)/5 + (\sin 7\omega t)/7 + \ldots.]$$

A computer generated plot of a square wave made up of the fundamental plus the first 20 odd harmonics shows the resulting wave is nearly a perfect square wave.



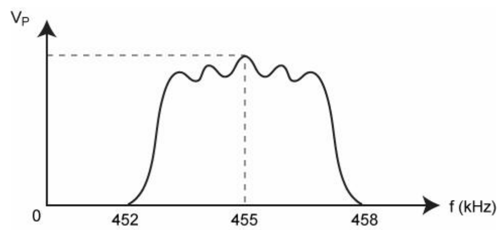
It can be shown that as the lower frequency harmonics are added it produces the flat top of the square wave and that the higher frequency harmonics shape the straight sides of the square wave.

In the frequency domain plot of the square wave, the amplitudes of the harmonics decrease in proportion to their frequency. The DC component is shown dashed.



Lab Procedures

1. Determine the calculated peak values of the first nine odd harmonics of a square wave when V_p = 10 V, f = 455 kHz, and t = 580 nSec. The time (580 nSec) is chosen to represent some point in time other than the peak value of the first harmonic. If you perform the calculations at the half wave time point, the voltage values would calculate to zero.
2. Record the values in table following this procedure.
3. Build the circuit following the procedure. Resistor values are determined by data sheet.
4. Connect channel 1 of the oscilloscope to monitor the input of the filter and apply a 10 V peak-to-peak 455 kHz sine wave.
5. Connect the oscilloscope channel 2 at TP_2 and measure V_{out}. Carefully fine tune the frequency to the maximum output voltage. There should be multiple peaks near the 455 kHz frequency. Make sure you are tuned to the slightly larger center peak.



6. Measure the peak values of V_{out} and V_{in}.
7. Determine the insertion loss or gain of the filter.
8. Set the function generator to produce a ± 5 V amplitude 455 kHz square wave. Fine tune the frequency of the function generator to peak the output voltage. Use the center peak frequency that causes the largest peak voltage at TP_2.
9. Measure the largest peak value at V_{out} at TP_2. Record the value in the table below.
10. Measure the frequency of the function generator using a frequency counter. Record the value in the table.
11. Determine the peak value of the third harmonic by setting the function generator at 1/3 of the 455 kHz value measured in step 5.
12. Use the center peak frequency and measure the voltage and frequency at TP_2. Record the values.
13. Repeat for the 5^{th}, 7^{th}, and 9^{th} harmonics by setting the function generator to one-fifth, one-seventh, and one-ninth of the fundamental frequency measured in step 5. Record the values.
14. Repeat for the even harmonics. Since a square wave is made up of a fundamental sine wave and odd harmonics, you can determine how "perfect" the function generator square wave is.
15. If a spectrum analyzer is available, produce the spectral display of the signal at TP_2 for square wave. Set the controls to display the amplitudes of the first nine harmonics.

Lab Questions

1. Compare the calculated values with the actual values. What may have caused any differences?

2. Sketch the frequency domain plot of the calculated values of the square wave.

3. Suppose the square wave is applied to a low-pass filter that exhibits a break frequency of 455 kHz and -20 dB/ decade roll-off. Assume the waveform has a fundamental frequency of 455 kHz and amplitude of 5 V.
   a. Calculate the amplitudes of the first nine harmonics which would be contained in the output signal of the low-pass filter.

   b. Sketch the frequency domain plot of the resulting output signal of the filter.

4. Draft a two to three paragraph conclusion based upon your calculations and measurements. Does the insertion loss or gain determination in Step 7 figure into your conclusion?

Peak Values of Harmonics

Formula: $y = V_p + 4V_p/\pi\ [\sin\omega t + (\sin 3\omega t)/3 + (\sin 5\omega t)/5 + (\sin 7\omega t)/7 + \ldots.]$

Square Wave Harmonic	Calculated Value	Actual Value	Frequency
First			
Third			
Fifth			
Seventh			
Ninth			
Eleventh			
Thirteenth			
Fifteenth			
Seventeenth			

Circuit Schematic