INTRODUCTION

We start our course with a quick review of systems of equations, diving right in the first day of class. In the classroom, use this first activity, as an ice breaker giving the students a chance to connect through a common exercise.

This is an old puzzle, a version of which appears in William Whiston's, “The Elements of Euclid” circa 1702. There is an answer, though a bit unexpected. The best part of this puzzle is that, with a little Algebra which is accessible by students of Introductory Algebra, we can prove that the distance between the earth and the rope would be the same if the original object were an orange or a soccer ball. How cool is that?

For a remote class, ask the students to complete the Why I love Algebra activity before class and use breakout rooms for discussion and comparing results.

For a class that is completely online, use the problem from “Extending the Lesson” or the Oreo Problem explained in “Practice Exercises”.

ACTIVITY FOR CONNECTION: WHY I LOVE ALGEBRA

The Question:

Imagine a rope is tied tightly around the Earth’s equator. Now an extra yard in length is added to the rope and the rope is repositioned so that every point on the rope is the same distance above the equator. Will the gap between the rope and Earth allow the passage of a soccer ball, an orange, or a thin knife blade?

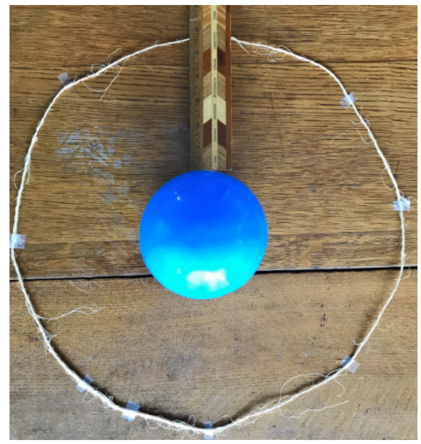
The process:

To test the students’ conclusion to the question above, start with object that can be held, any type of ball will do.

1. Take a round object, wrap a long string tightly around the circumference.

2. Remove the string, add a yard to its length, and cut the string.

3. Reposition the string around the object so every point on the string is the same distance from the outer edge of the object.
4. Measure and record the distance between the outer edge of the object and the string.
5. Repeat the process with multiple round objects of varying sizes
6. Compare the results.



Materials list:

For Each Group

- ball of string,
- scissors,
- yard stick, ruler, or tape measure
- round objects of varying size (marble, golf ball, tennis ball, whiffle ball, soccer ball, beach ball)

For Each Student

- Why I Love Algebra (see Appendix)

Discussion Questions

What conclusions can you draw from the results of our table? Can you answer the original question with certainty - Will the gap between the rope and Earth allow the passage of a soccer ball, an orange, or a thin knife blade?

To answer this question with certainty we use systems of linear equations. Review the process of solving systems of equations and come back to this question at the conclusion of class.

For a class taught completely online, ask your students post their ideas on how they would support their conclusion with an algebraic explanation.

The algebra behind the activity

Identifying the variables

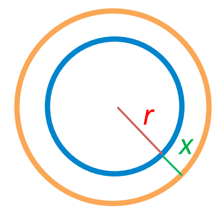
- r is the radius of the earth and
- x is the distance from the outside edge of the earth to the string
- y is the length of the string

Setting up the equations

The circumference of any object is $C = 2\pi r$

The length of the string is the circumference of the earth plus one yard (36 inches),

$$y = 2\pi r + 36$$



The length of the string can also be defined as the circumference of the circle formed by the string, where $r + x$ is the new radius, so

$$y = 2\pi(r + x)$$

Solving the system of equations

Substituting the length of the string from the first equation into the second we get:

$$2\pi r + 36 = 2\pi(r + x)$$

Now we will solve for x:

$$2\pi r + 36 = 2\pi r + 2\pi x$$

$$36 = 2\pi x$$

$$\frac{36}{2\pi} = x$$

$$5.73 = x$$

Conclusion:

The distance between a round object of any size and the string around it will always be the same, approximately 5¾ inches.

EXTENDING THE LESSON: JACK REACHER

Introduction:

For the next discussion point in class, use this excerpt from one of Lee Child's books in the Jack Reacher series (Die Trying, Lee Child pg. 536). You can use this to start the class session on applications of linear systems. You can also use it as a group work.

It can be used similarly for a remote class, using breakout rooms for discussion, and comparing results.

For a class that is completely online, assign the problem handout and ask the students to submit their work individually.

The Process:

After reading the excerpt, Distribute the handout and ask the student to solve this problem algebraically to confirm Jack Reacher's trial and error logic.

PlayStation Europe. "Jack Reacher." *Flickr*, Yahoo!, 15 Aug. 2013, www.flickr.com/photos/playstationblogeurope/9517191406/in/dateposted/.

Reacher closed his eyes and started to calculate. It was like being back in grade school. He's two hundred miles ahead, doing fifty miles an hour. You're chasing him at a hundred and sixty. How long before you catch him? Grade school math had been OK for Reacher. So had fighting in the yard. The fighting part had stayed with him better than the math. He was sure there must be some kind of a formula for it. Something with x and y all over the damn page. Something equaling something else.

But if there was a formula, he had long ago forgotten it. So he had to do it by trial and error. Another hour, Stevie would be two hundred and fifty miles from home. The Night Hawk would have done one hundred and sixty. Way behind. An hour after that, Stevie would be three hundred miles out, and the Night Hawk would be three hundred and twenty. Overshot. Therefore, they were going to catch him somewhere near the top of the second hour. If they were headed in the right direction.

Materials list:

For Each Student

- Jack Reacher student handout (see Appendix)

The algebra behind the activity

Identifying the variables

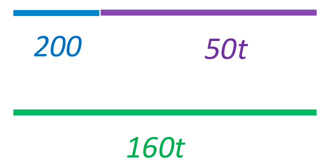
- t is the time they travel in hours
- d is the total distance they travel

Setting up the equations

When Jack caught up with Stevie, they have traveled the same total distance.

For Stevie

$$d = 200 + 50t$$



For Jack

$$d = 160\,t$$

Solving the system of equations

Substituting the distance from the first equation into the second we get:

$$200 + 50t = 160\,t$$

$$200 = 110\,t$$

$$\frac{200}{110} = t$$

$$1.8 = t$$

Conclusion:

Jack will catch Stevie in about one hour and 48 minutes.

PRACTICE EXERCISES

Oreo Problem

There are 53 calories in one original Oreo cookie and 70 calories in the Double Stuf version. Find the number of calories in one wafer and one layer of filling. (Double stuf Oreo has twice the filling of the original Oreo)

Identifying the variables

- x is the calorie of one wafer
- y is the calorie of one layer of filling

Setting Up

$$2x + y = 53$$

$$2x + 2y = 70$$

Final Answer

There are 18 calories in one wafer and 17 calories in one layer of filling

Oreo Problem as an activity

This first Practice Exercise can be used for group work as well. Separate the class into groups by asking how the students eat an Oreo cookie – separate the cookie and eat the filling first, only eat the filling, only eat the wafer, taking bites of both the filling and the wafer, never ate an Oreo . For the groups that are too large, separate those groups further by asking if they eat the cookie alone, with milk, by dunking it in milk.

Give the students the problem as written above or provide the nutritional labels and ask them to find the number of calories, (or grams of sugar, grams of fat) in one wafer and one single layer of filling. There is a bit of critical thinking here at the start of the setup – why does a double stuff have fewer calories than the original version? Remind students that they have to compare the same number of units. (Note that the labels give the values for different number of cookies)

After finding the calories, ask the students to find out how much more filling is in a Mega Stuff version?

Answer: The calorie difference with double stuff is $90 - 53 = 37$ calories, which comes from the filling, so $\frac{37}{17} \cong 2.2$, it has 2.2 times more filling compared to the original one.

Emergency Relief

Bottled water and medical supplies are to be shipped to victims of a hurricane by plane. Each plane can carry 90,000 pounds and a total volume of 6000 cubic feet. The bottled water weighs 20 pounds per container and measures 1 cubic foot. The medical kits each weigh 10 pounds and measure 2 cubic feet. How many containers of bottled water and how many medical kits can be sent on each plane?

Identifying the variables

- *x* is the number of bottled water containers
- *y* is the number of medical kits

Setting Up

$$20x + 10y = 90000$$

$$x + 2y = 6000$$

Final Answer

Each plane can carry 4000 bottled water containers and 1000 medical kits.

Kayaking

It is 6 miles in a kayak to the Fish Islands from my house. The trip to the island takes 2 hours traveling against the current and 1¼ hours for the return trip (with the current). How fast can I move the Kayak is there was no current? The answer can be rounded to the nearest hundredth.

Identifying the variables

- *x* is the speed of the kayak
- *y* is the speed of the current

Setting Up

$$2(x - y) = 6$$

$$1.25(x + y) = 6$$

Final Answer
The kayak moves at 3.9 miles per hour. (The speed of the current is 0.9 mph).

Milk fat Percentage
Prof. Glatt likes 2% milk (2% fat) for her cereal in the morning. Her parents only buy whole milk (3.5% fat) and non-fat milk (0% fat). While she is visiting her parents, how much of each type of milk does she need to mix to get 3 cups of 2% milk. The answer can be rounded to the nearest tenth.

Identifying the variables
- *x* is the number of cups she needs from 3.5% milk
- *y* is the number of cups she needs from 0% milk

Setting Up
$$x + y = 3$$
$$0.035x = 0.06$$

Final Answer
She needs 1.7 cups of whole milk and 1.3 cups of non-fat milk.

Buying a Car
To buy a car for $36,000, Stacy uses her credit card which charges 20% interest and borrows the rest from an online bank that charges 4.5% interest. If she pays $2178 in interest the first year, how much did she put on the credit card?

Identifying the variables
- *x* is the $ amount put on the credit card
- *y* is the $ amount borrowed from the online bank

Setting Up
$$x + y = 36000$$
$$0.2x + 0.045y = 2178$$

Final Answer
Stacy put $3600. On her credit card (borrowed %32,400 from the bank)

Bonus Question


Identifying the variables

- *x* number of dimes
- *y* number of quarters

Setting Up

$$x + y = 20$$

$$0.1x + 0.25y + 0.9 = 0.25x + 0.1y$$

Final Answer

The man has 13 dimes and 7 quarters

ARTICLE AND VIDEOS LINKS

Cool Careers in Science – PBS Learning Media (quality control in drugs) https://cptv.pbslearningmedia.org/resource/biot09.biotech.car.qcbiot/career-profile-quality-control-microbiologist/

LIST OF STUDENT HANDOUTS

- Why I love Algebra
- Jack Reacher
- Oreo Activity
- Practice Exercises