Lesson 6: Pouring Concrete

PREREQUISITE ASSUMPTIONS

Before beginning this lesson, students should

- Understand the need to calculate cylinder volumes for pier footings (decks etc.)
- Know the definitions of radius, diameter and pi.
- Know concrete is ordered in cubic yards

Commented [CP1]: We are also assuming they know what radius, diameterm and pi are based on their work in Lesson 4.

COMPETENCIES

Unit 1. Use of a Scientific Calculator

B. Apply basic mathematical operations in solving word problems

B.1. you translate a verbally stated problem into performing an equivalent computation

B.2. you interpret the computed answer to a word problem **C. Perform calculations using the**

Unit 5. Measurement

A. Perform calculations with quantities having units of measure: inches, inches squared, inches cubed, feet, feet squared and feet cubed only.

A.1. you perform addition, subtraction, multiplication, division, exponentiation to powers of 2 and 3, square root taking or combinations of these operations for quantities expressed as measurements

A.2. you expresses the answer with the appropriate units

A.3. you use a scientific calculator to compute the answer to problems involving measured quantities

B.4. you convert volume measurements to different square and cubic units of length measure

Unit 6. Applied Plane Geometry

C. Perform calculations using the Pythagorean Theorem

C.2. you determine the height of an isosceles triangle with given sides by dropping a perpendicular and using the Pythagorean Theorem

C.3. you use a construction calculator to perform calculations involving the Pythagorean Theorem

D. Calculate perimeters and areas of closed planar figures

D.7. you compute the area of a circle given its radius or diameter

D.8. you compute the circumference of a circle given its radius or diameter

MATERIALS

- Geometry formula sheet
- Example volume box with 27 cubes
- Crush test cylinder of concrete (optional)
- Foam cubes that are the following dimensions:
    - 1 cubic inch
    - 1 cubic foot
    - 1 cubic yard

- Blueprint of one of the sheds that requires an 8’ x 10’ concrete slab

VIDEOS for 3-ACT:

- Popcorn Picker: http://threeacts.mrmeyer.com/popcornpicker/

Notes to Self

- One thing I want to do during this lesson ….
- One thing I want to pay attention in my students’ thinking …
- One connection or idea I want to remember …

Suggested Timeline

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
5 minutes	Work Question #1	Individual Work
8 minutes	Discuss Question #1	Instructor Led
4 minutes	Act 1, questions 2 - 3	Small groups
10 minutes	Discuss 2 & 3, answer #4 and #5	Whole class
2 minutes	Act 2, Question 6	Small groups
4 minutes	Discuss Question 6	Whole class
12 minutes	Question 7	Small groups
4 minutes	Discuss Question 7 and Act 3 Reveal	Whole class
4 minutes	Question 8	Individual Work
12 minutes	Question 9	Small groups
4 minutes	Discuss Question 9	Whole Class
BREAK (69 minutes)		
8 minutes	Work Question 10	1 -2 minutes individually then Whole class
10 minutes	Work Questions 11 and 12	Small Groups
6 minutes	Discuss Questions 11 and 12	Whole Class

END DAY 1		
3 minutes	Work Question 13	1 -2 minutes individually then Whole class
12 minutes	Work Questions 14 - 16	Small Groups
6 minutes	Discuss Questions 14 - 16	Whole Class
3 minutes	Making Connections	Whole Class

[Student Handout]

SPECIFIC OBJECTIVES

By the end of this lesson, you should understand

- The following math concepts related to circles: Circumference, radius, diameter, and formulas for area and circumference.
- The following math concepts related to volumes: Volume formula for a cylinder and calculating the volume of a rectangular solid
- That materials such as concrete, gravel, soil and sand are ordered in cubic units (usually cubic yards or "yards".)

By the end of this lesson, you should be able to

- Calculate the missing side of a right triangle using the Construction Master Calculator
- Calculate Volume for right cylindrical columns.
- Convert units from cubic inches and cubic feet to cubic yards

PROBLEM SITUATION #1 : Pythagorean Theorem on the Construction Master Calculator

Before moving on to the next topic, we're going to spend a few more minutes on the Pythagorean Theorem because it is so important in construction. Take out your homework from Lesson 5 (the previous lesson), and use it to answer Question 1 below.

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
5 minutes	Work Question #1	Individual work

8 minutes	Review Question #1	Instructor Led

Notes: The point of this problem is to show students how to use the Construction Master Calculator to find the 3rd side of a right triangle. It solves the Pythagorean Theorem for the students.

Cruise the room while students take out their homework and do question #1. Be sure students are writing down the steps of solving using a square root. Be sure to pause and speak to students that don't have their homework done and encourage them to keep on top of the homework.

If students are struggling, or are done early, ask them to check with other folks in their group to either ask for help or offer help.

Look for a student that has really clear work, ask to have them put their work on the board or show it on the Doc Cam.

ASK: ***"Has anyone done this work before on the calculator using the 'Rise' 'Run' and 'Diag' keys?"***
If a student says yes, ask them to come up and demonstrate on the Doc Cam. Draw the triangle and label it 'Rise' 'Run' 'Diag' and 'a' 'b' 'c' while they work.
If not…
DEMONSTRATE: Solve the problem using the Construction Master Calculator

Tell students: ***"Diagonal is the hypotenuse. It is always the longest side."***
ASK: ***"Why did I make you use the square root to solve the formula when you could have just used these shortcut keys?"***
Answers that need to be highlighted:

- Needed for other formulas (like finding the radius if you know the area of a circle)
- Now you know what the calculator does when you push those 3 buttons.

Pg 129 8
Pg 130 11
Pg 131 19

Tell students: ***"now check your work on these other two problems from your Lesson 5 homework using the shortcut method on the calculator."***

1. Show your work for problem # 8 on page 129 from the Lesson 5 Homework Below

Answer:

Side note: somewhere I want to show another way to square a wall if you have a rectangle or a square is to use the property that the diagonals must be the same length.

ALSO bring up the property of dropping a perpendicular in an isosceles triangle Also that having equal side lengths is not enough. That could be a parallelogram or a rhombus.

PROBLEM SITUATION #2 : Popcorn Picker Task

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
4 minutes 10 minutes	Act 1, questions 2 and 3 Discuss 2 & 3, answer #4 and 5	Small groups Whole class

Notes: The 3-act math task is an opportunity to introduce finding the volume of a cylinder in a fun context with a 'low entry.' Act 1 is a 30 second video that invites students to wonder about the cylinders filled with popcorn. Act 2 is their opportunity to decide what they need to figure out which cylinder holds more popcorn, and then, once armed with what they need, to actually figure it out. Act 3 is simply the 'reveal' where they get to watch a 10 second video to see if they are correct.

Start by showing the video.
After you show the video, tell students ***"Take a minute to write down on your paper everything you notice (that is question 2) and everything you wonder (that is question 1). After a minute is up, share with your group your answers. Then we'll share out as a whole class."***

Don't give students too much time for 2 and 3. Maybe only give 30 seconds before telling them to share with their groups, and then just a minute or two before sharing out as a whole class.
Ask groups to share (1 item per group) and then extras from anyone for each of the questions.

NOTE: It is important to value each noticing and each question that arises. Be sure to save time at the end of class to go back and at least acknowledge, if not answer, all of the wonderings...

After sharing all of the wonderings, ask students to vote on the question that is most interesting to them. If it is one that doesn't involve math, just try to answer it outright.

Have students write down the question they are going to figure out the answer to in the space under #4.
Tell students: ***"Now that we have a question to answer (which one holds more, or do they hold the same amount), I want you to talk in their groups and take an educated guess at the answer WITHOUT DOING ANY CALCULATIONS. It could be gut feel. It could be based on only what they see. It should take LESS THAN 30 SECONDS."***
Put a table up on the board and have each group share their prediction. SAVE it to come back at the end and give a quick 'good job' to the ones that were correct/closest. I do that by having everyone in the room do a single clap.

ACT 1:

Show the act 1 video

2. What do you notice?

Some possible things students might notice: -both are cylinders -one is tall and skinny, the other is short and wide -both are made from an 8 ½" x 11" sheet of paper -Popcorn...

3. What do you wonder?

Some possible things students might wonder: -which container will hold more? -do they both hold the same amount? -does the paper overlap at the seam? -how long does it take to fill them up?

4. What question does the class agree to work on?

Need to support the class agreeing on the question 'which will hold more?' OR 'do they both hold the same amount?'

5. Make your prediction

Predictions possible: - The tall one holds more popcorn - The short one holds more popcorn - They both hold the same amount ← *this is the most common guess*

ACT 2:

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or

		individual work)
2 minutes 4 minutes	Question #6 Review Question #6	Small Group Whole Class
Notes: The point of this problem is to really get students to think about what problem solving actually is. In the real world, you are not given everything you need to solve a problem, you have to start by figuring out *what the problem is*, and then figuring out *what you need to solve the problem*. Give students a couple of minutes (at most) to brainstorm in their groups about what they need. Provide a formula sheet and have them choose the cylinder volume formula. Draw the image of the cylinder on the board and label it with the variables		

6. What do you need to answer the question?

NEED:
-Size of paper (they likely will guess that it is 8 ½" x 11" - which is correct)
-Formula for the volume of a cylinder: $V = \pi r^2 \mathrm{h}$

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
12 minutes 4 minutes 4 minutes	Question #7 Review Question #7 and Act 3 reveal. Answer #8	Small Group Whole Class Individual work
Notes: The point of this problem is to give them a chance to work on the volume. This is NOT an easy problem, so cruising the room and keeping an eye out for frustrated students is KEY. Tell students to start working on #7. Students work in their groups to apply the formula to the problem for both cylinders. AFTER THE STUDENTS STRUGGLE FOR A FEW MINUTES, SEE IF ANY OF THE STUDENTS HAVE NOTICED THAT THEY ALSO NEED THE CIRCUMFERENCE FORMULA IN ORDER TO CALCULATE THE AREA. Hopefully at least one student figures it out after a couple of minutes. At that point, stop the class and have a whole class discussion asking if they have enough information to figure out r and h. THEN, have a student explain how you also need the circumference and also put the formula up on the board.		

$$C = 2\pi r$$

Draw a circle and label C and r.

NOTE: Some students will want to measure the radius. ALLOW that as a verification. Or, if they measure first, make sure they go back and use the circumference.

As students are finishing up #7, have two students put up the work for both cylinders on the board. Don't spend too much time going over them, just make sure they get into the notes.

Then, show the video for Act 3 and have students answer #8. #8 is an opportunity for students to reflect. Either skip going over it completely, or write the key reflections as a 'making connections' summary.

Point out/show the fact that the r^2 dominates the problem, so the one with the bigger radius will usually win.

7. Answer the question and show or explain how you figured it out

Answer:
Tall skinny - radius calculation… C = 8.5", so radius is 8.5/(2*pi)... radius = 1.3528"
Volume calculation… V = pi*(1.3528)^2*11 so V = 63.2 cubic inches
Short and wide - radius calculation… C = 11", so radius is 11/(2*pi)... radius = 1.7507"
Volume calculation… V = pi*(1.7507)^2*8.5 so V = 81.8 cubic inches

ANSWER: short one holds more.

Commented [CP2]: Concern == students with poor algebra/ formula manipulation skills may really get stuck here and may proceed with no idea how radius of the circle was actually calculated.

Commented [CP3R2]: Could students use a piece of paper and ruler/tape measure to determine radius?

ACT 3:

Show the Act 3 video

8. Take a minute to reflect on what you learned. Did your answer match your prediction? Why or why not? Write down everything you learned below.

No correct answer here. Just encourage students to reflect and write at least one thing
Likely not matching prediction.
Probable answer: Yes.

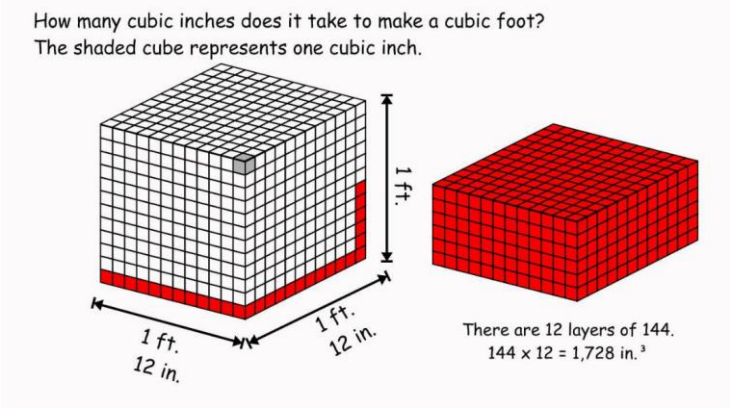
Duration	Activity (Indicate question number)	Suggested Structure (Indicate group, whole class or individual work)
12 minutes 4 minutes	Question #9 Review Question #9	Small Group Whole Class

Notes: The point of this problem is to have students convert the units by hand, so they really get that you can't just divide by 12 to get from cubic inches to cubic feet. Each dimension needs to be the correct units.

Tell students: ***"I know we've done work where you calculator converts units for you, but I want you to convert the units of the larger volume cylinder from cubic inches to cubic feet without using your units buttons on your calculator (or do it on your cell phone calculator). Discuss with your group how to do it before you begin. AFTER you do that (9a), use your unit conversion trick on your calculator to check your work."***

Have a student put the work up on the board for 9a
Demonstrate (or have a student walk it through while showing on the doc cam) for 9b
When reviewing this part c as a class, be sure to ask 'why is the cubic feet so small? Does that make sense?'

Maybe use rulers to demonstrate the size of a cubic foot on the desk and put the paper tube in the cube so students can see how little space it takes up.



9. Convert the units in your answer to the units your instructor tells you below:

a) Set up the calculation by hand and then type it into your calculator. Write down what you typed into your calculator below.

One method: 81.8 cubic inches… since 1 foot = 12 inches, 1 square foot = 12in x 12 in = 144 square inches, and 1 cubic foot is 1 foot x 1 square foot, so = 12 in x 144 square inches = 1,728 cubic inches. DIVIDE 81.8 cubic inches by 1,728 cubic inches = 0.047 cubic feet.

b) Now use your calculator function to do the unit conversion for you.

Students will type in 81.8 and then hit the 'INCH' button 3 times. Then, they hit the 'FEET' button once and it will convert to 0.047 cubic feet

c) Do your answers match? If not, why not?

PROBLEM SITUATION #3 : Concrete Columns

You're building a deck. The building code for your area says that a deck of the size you are building needs to have 6 posts to support the deck. Each post needs to sit on a concrete base. According to code, the bottom of the supporting column must be at least 4' below the ground to get below the frost line. You'll use six Sonotubes to form the six concrete bases you'll need to pour.

According to common residential deck building practice, the supporting columns typically have a diameter of 12".

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
8 minutes 10 minutes	Work Question 10 Work Questions 11 & 12	1 -2 minutes individually then Whole class Small Groups

6 minutes	Review Questions 11 & 12	Whole Class

Notes: Question 10 is likely too hard for anyone to get individually, but give them a minute to try before opening it up to a class discussion. Support reasonable ideas. Let students know it is ok to not have a strategy for this.
If nobody has any ideas then offer a hint… "***Is there an easier shape to work with than a circle/cylinder that you could use instead to do your estimate?"***
Don't offer the sophisticated estimate if nobody comes up with it.

Before moving on to question 11, make sure that everyone has the formula sheet out. Ask the students ***"which formula will you need to use for this problem situation?"***

NOTE for Question 11: If students use 6 inches for the radius, the calculator will spit out cubic yards. BUT, if the students use 0.5 feet for the radius, the calculator will spit out cubic feet. So, allow students to do either for #11. In #12, they will convert to the other one.

Allow them to use their calculators for #11. Walk around while they are working and see what people are typing in. Ask students to write down what they typed into their calculators for #11 along with their answer.
During the class discussion ask why we chose to round to 2 decimal places. Ask: ***"What would you have done if the problem didn't specify how to round your answer?"***

10. Spend a minute thinking on your own… what strategy could use to do a quick estimate of the total concrete you'll need for the job?
    a. Write down your strategy below.

    b. Now use the strategy to estimate the amount of concrete you'll need.

Possible Answers:
Simple… pretend it is a square with 1 foot sides. Then, the volume is 1' x 1' x 4' or 4 cubic feet per post. X 6 = 24 cubic feet.

Sophisticated estimate: Area of a circle is pi*r^2 and the radius is 6" or ½ of a foot. So, rounding…

3 * ½ * ½ is an area of ¾ . Multiply that by the height and by 6 posts… ¾ * 4 * 6… about 18 cubic feet.

Volume of one post (in cubic feet): V = pi*(6/12)^2*4

11. Determine the total amount of concrete you need to order for the job. An important consideration when placing your order is that the ground will not be perfectly level. To address that, you will need to do the standard practice of most contractors. You will need to calculate the total volume needed and then add 10% to the total concrete order. Round your answer to the nearest hundredth (2 decimal places).

Commented [SHBP4]: Should this be moved to AFTER the conversion question? The answer is the same either way.

Answer: **0.77 cubic yards OR 20.7 cubic feet**
Volume of one post calculated *on the construction master*: V = pi*(6")^2*4'

One post: 0.116355 cubic yards or 3.14159 cubic feet
Six posts: 0.698132 cubic yards or 18.84956 cubic feet
0.698132 * 10% = 0.0698132 cubic yards or 20.73 cubic feet
Total order in cubic yards is 0.767945
This is the spot where Sandy's cubic yard box should be brought out.

12. Now, convert the units for the total amount of concrete. If your answer in #11 was in cubic yards, convert it to cubic feet. If your answer was in cubic feet, convert it to cubic yards.
    a. First do the conversion *without using your units button on your calculator* Set up the calculation by hand and then type it into your calculator. Write down what you typed into your calculator below:

    b. Check your answer by using the units button on your calculator to make sure you are correct. If not, go back to part a and fix your work.

Convert to cubic feet: 3 ft = 1 yards… so 3 ft x 3 ft x 3 ft = 1 yd x 1 yd x 1 yd… so 27 cubic feet = 1 cubic yard.

Unrounded: 0.767945 cubic yards * 27 cubic feet = 20.7 cubic feet

Rounded: 0.77 cubic yards * 27 cubic feet = 20.8 cubic feet of concrete should be ordered.

Convert to cubic yards: 27 cubic feet = 1 cubic yard:
Unrounded: 20.73451 cubic feet / 27 cubic feet = 0.767945 or 0.77 cubic yards
Rounded: 20.7 cubic feet /27 cubic feet = 0.766667 or 0.77 cubic yards

PROBLEM SITUATION #4 : Pouring a Concrete Slab for a Shed

Exciting news! There is a buyer for one of the sheds out back. The new potential owners need help determining additional costs. How much will the concrete cost for the concrete slab they need to pour?

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
3 minutes 12 minutes 6 minutes	Work Question 13 Work Questions 14 - 16 Review Questions 14 & 16	1 -2 minutes individually then Whole class Small Groups Whole Class

Notes: This problem situation is to deal with the situation where the thickness of the concrete is in inches, but the other dimensions are in feet. The shape is fairly straightforward, since the units are meant to be the challenge.

ANNOUNCEMENT TO THE STUDENTS: ***"you will be looking back at Lesson 1 in the Lesson 7 classwork, so please be sure to bring the lesson 1 handout to class"***

Allow students just a couple of minutes to talk about #13 and then go around the room and have each group share out one item, then get all ideas up on the board.

AFTER #13 is answered, hand out the shed footprint with the dimensions on it. Tell the students: ***"You will be using 2x4's standing on their sides as a border to hold the concrete in when you pour the slab. You will pour the slab so it is flush with the tops of the 2x4's so you can smooth it out easily."***

Since they have now had quite a bit of practice with volumes and units, this problem situation is set up so that the groups can basically work on their own. Pay special attention to the weaker students and consider pulling them together to a side group to help them through the problem if they are looking frustrated.

In problem #14, the students need to realize that they need:

- Formula for the volume of a rectangular solid

- Figure out the units for volume that go with the price
- Figure out how to get the volume units to match the needed units
- They were not told to add 10%. Allow them to choose whether or not to do it themselves. Either way is correct for #14 and makes no difference for the final price.

In problem #15, the students need to realize that they need to:

- Convert the volume units from cubic yards to cubic feet
- You can only buy whole bags of concrete
- They were not told to add 10%. If they do, it adds 4 more bags. Allow them to choose whether or not to do it themselves. Either way is acceptable.

#16 is really preference and how much they value their own time.

When students are finishing up, cruise the room and look for (ideally) two or more different ways that the students approached the calculations for #14. Have the students with the different methods put them up on the board.

When groups are working on #15, check in to see how many get tripped up on changing things into cubic feet. If they don't do it, their answer will be really far off (they'll only need 2 bags). So, if students stop early, be sure to check in that their answer is in the ballpark. If it isn't, don't point out their mistake, ask them ***"does that answer seem reasonable to you based on what you got for #14?"***

13. Talk to you group. What do you need to know to figure out the cost?

Possible Answers:

- The dimensions of the shed
- How level is the ground
- How deep do we need to pour the concrete
- Cost of the concrete

Students are given the ACTUAL footprint of one of the sheds out back.

14. Determine the total cost of 3000 psi concrete you would order in cubic yards if you order the concrete from Wingra Stone (https://www.wingrastone.com/wingra-redi-mix/pricing/)

Total cost: $277.50

Slab dimensions... 8' x 10' by 3.5" (height of a 2x4) = 0.864198 cubic yards
Add 10%… 0.864 + 0.086 = 0.95 cubic yards. Minimum purchase is 1 yard.
$152.50 for 1 yard of 3000 psi concrete + $125.00 minimum delivery charge = $277.50

15. Another option to purchase the concrete is to buy it in bags. If 1 bag of concrete costs $3.99 and covers 2/3 of a cubic foot, determine the total number of bags you would order and the cost (assume no tax).

Total cost: $155.61

Slab dimensions... 8' x 10' by 3.5" (height of a 2x4) = 0.864198 cubic yards
0.864198 cubic yards = 23.33 cubic feet
Add 10%… 23.33 cubic feet + 2.33 cubic feet = 25.7 cubic feet
1 bag of concrete = 2/3 of a cubic foot
25.7 cubic feet/.67 cubic feet/bag = 38.4, round to 39
39 bags of concrete = $3.99 * 39 =$155.61

16. Which way would you go... bags or truck and Why?

There is no correct answer here. Yes, the bags cost less, but the effort to mix 35 bags could be a large enough deterrent.

MAKING CONNECTIONS

-Formula for the volume of a cylinder: $V = \pi r^2 h$
-Circumference is $C = 2\pi r$
-Carpenters must know how much concrete to order for piers, flatwork and other concrete applications on a job.

-It is common for columns and piers to be cylindrical

Practice/Homework

Pg 105 1, 3, 4, 6
Pg 106 11, 17
Pg 113 3
Pg 113 4 (extra credit)
Pg 115 13 (extra credit)