Lesson	***Objectives***	***Material***
5.1	System of equations	Gym membership
5.2	System of equations	Shapes
5.3	Systems of Equations: Graphing method	Graphing Method
5.4	Systems of Equations: Substitution method	Substitution method
5.5	Systems of Equations: Elimination method	Elimination method
5.6	Systems of Equations: Cramer's Rule	Cramer's Rule

Prerequisite Assumptions
Before beginning the lesson, students should understand and be able to apply;

- Kirchhoff's Voltage Law
- Kirchhoff's Current Law
- Ohm's Law
- Power Rule

Specific Objectives
By the end of this lesson, you should understand;

✓ Graphing Linear Equations
✓ Solving a system of equations

By the end of this lesson, you should be able to;

✓ Use the graphing method to find the solution of a Linear System of Equations
✓ Use the substitution method to find the solution of a Linear System of Equations
✓ Use the elimination method to find the solution of a Linear System of Equations
✓ Use the Cramer's rule to find the solution of a Linear System of Equations
✓ Graph non-linear equations
✓ Use the quadratic equation to solve second order equations

<u>Problem Situation 5.1 – Gym Membership</u>

1) You find this flyer and think you want to start working out. Pick the number of months that you want to try working out. Predict which plan that you think will work for you, based solely on cost (because you just do not have a lot of money), using the number of months you chose.

2) Write equations for the costs of each of the three scenarios based on any number of months you chose. *(the number of months is the independent variable)*

3) Which two of these equations are linear equations?

4) Graph each of the linear equations on the following graph. You might want to use a table.

5) Is there a point in time in which the costs of the first two plans are the same? This is the **point of the intersection**, (x,y). This *ordered pair* of numbers is the *solution* for each of the equations.

A **system of equations** is a set of equations with *two or more* variables that represent a *single* situation. The two variables, months and costs, are in both equations. The solution to the point of intersection is the <u>number of months</u> that result in the <u>same</u> cost.

- ✓ **One** unknown variable (solution) can be found with a *minimum* of **one** linear equation.
- ✓ **Two** unknown variables can be found with a *minimum* of **two** linear equations.
- ✓ **Three** unknown variables can be found with a *minimum* of **three** linear equations.
- ✓ Examples of a system of equations:

$x + y = 6$ $\qquad$ $a + 2b = 4$

$x - 2y = 6$ $\qquad$ $2a + b = 5$

How could you find the point of intersection for these two examples?

Problem Situation 5.2 – Shapes

1) Shapes Act 1: View this video Shapes. What is happening?

2) What would you guess each shape is worth?

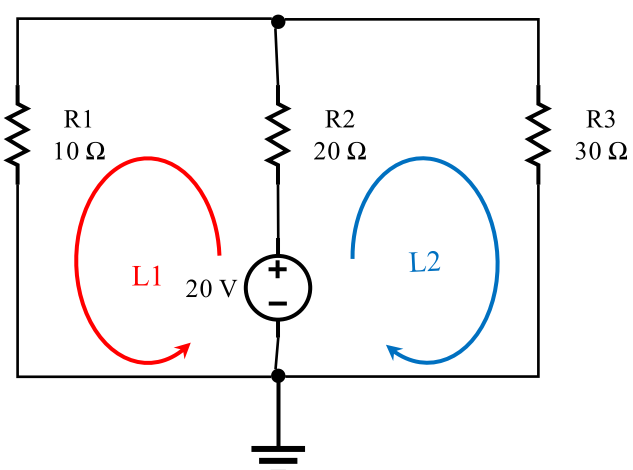
3) What information do you need to determine the points value assigned to each shape?

4) Determine the points assigned to each shape.

Problem Situation 5.3 – Application problem - Graphing method

Mesh analysis uses Kirchhoff's Voltage Law and Ohm's Law. The results are typically a system of equations.

1) Graph the following linear equations to determine the solution for the currents. Make sure to label the graph well.



Derivation of the Mesh Equations

$L1\text{:}\ 0 = 20V - V_{R2} - V_{R1}$

$0 = 20V - (I_1 + I_2)R_2 - I_1R_1$

$30I_1 + 20I_2 = 20V$

$3I_1 + 2I_2 = 2V$

$L2\text{:}\ 0 = 20V - V_{R2} - V_{R1}$

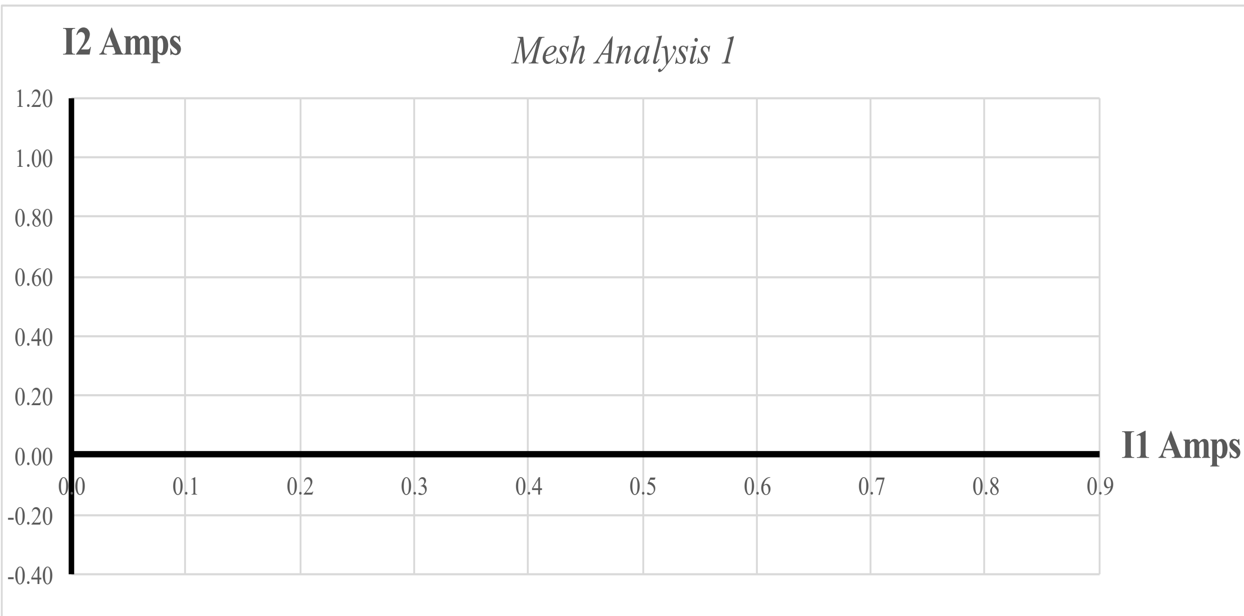
$0 = 20V - (I_1 + I_2)R_2 - I_2R_3$

$20I_1 + 50I_2 = 20V$

$2I_1 + 5I_2 = 2V$

$3I_1 + 2I_2 = 2$	
I_1	I_2

$2I_1 + 5I_2 = 2$	
I_1	I_2



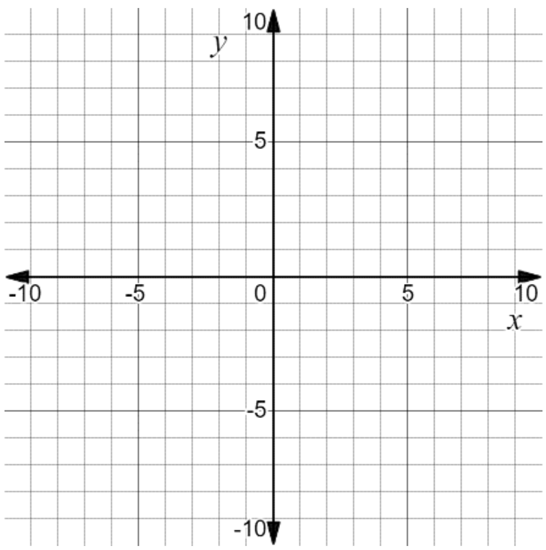
2) The graphical solution is the ordered pair (x, y) at the intersection. Identify the intersection point..

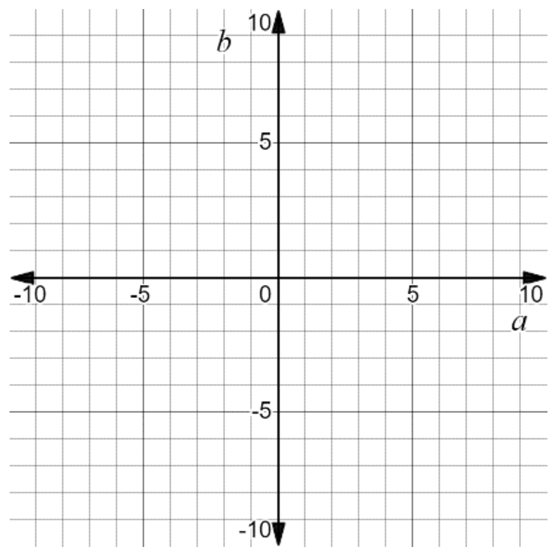
3) Verify that the solution is correct by applying Kirchhoff's Voltage & Current Laws to the circuit.

4) Build your expertise and confidence by solving the following two systems of equations using the *graphical* method.

a)
$x + y = 6$
$x - 2y = 6$

b)
$a + 2b = 4$
$2a + b = 5$





Problem Situation 5.4 – Substitution Method

Another method for solving a linear system of equations is the substitution method.
To solve using the substitution method:

- "Solve" one of the equations by isolating one variable on one side of the equality in terms of the other variable. Often you will pick the variable that 'looks' like it would be easiest to isolate.
- Substitute that isolated variable into the other equation and then solve for the remaining variable.

Example:

$Eqn\ 1: 4x - 5y = 6$
$Eqn\ 2: x + 2y = 8$

$$Eqn\ 1:\ 4x - 5y = 6 \quad \rightarrow \quad 4x = 6 + 5y \quad \rightarrow \quad x = \frac{6 + 5y}{4} \rightarrow \text{x is isolated in terms of y}$$

$$Eqn\ 2:\ x + 2y = 8 \quad \text{substitute for x:} \quad \left(\frac{6 + 5y}{4}\right) + 2y = 8 \ \rightarrow\ (6 + 5y) + 4 * 2y = 4 * 8$$

$$6 + 5y + 8y = 32 \quad \rightarrow \quad 6 + 13y = 32 \quad \rightarrow \quad 13y = 32 - 6 \quad \rightarrow \quad y = \frac{26}{13} \quad \rightarrow \quad \boldsymbol{y = 2}$$

Substitute value for y back into original equation:

$$4x - 5 * 2 = 6 \ \rightarrow \ 4x - 10 = 6 \ \rightarrow \ 4x = 16 \ \rightarrow \ x = \frac{16}{4} \ \rightarrow \ \boldsymbol{x = 4}$$

Check the solution: The ordered pair (4, 2) should make **both** equations true.

$Eqn\ 1: (4 * 4) - (5 * 2) = \mathbf{6}$ √
$Eqn\ 2: 4 + (2 * 2) = \mathbf{8}$ √

1) Use substitution to determine the solution for the following systems of equations.

a) $3x - 2y = 6$
$2x - 3y = 4$

b) $5a + b = 15$
$a + 5b = 27$

c) $x + y = 6$
$x - 2y = 6$

d) $a + 2b = 4$
$2a + b = 5$

2) Using the substitution method, solve for the currents in our **initial** circuit.

$3I_1 + 2I_2 = 2$
$2I_1 + 5I_2 = 2$

Problem Situation 5.5 – Elimination Method (addition method)

Another method for solving a linear system of equations with two equations and two unknowns is called the **Elimination Method**. To solve using the elimination method;

- Set up the equations so that when you add them together one of the variables is eliminated.
- Determine the value of the remaining variable
- Insert the solution for the solved variable into one of the equations to find the other variable.

EXAMPLE:

$Eqn\ 1\text{:}\ \ 4x - 5y = 6$
$Eqn\ 2\text{:}\ \ x + 2y = 8$

- "Change" Equation 2

$Eqn\ 2\text{:}\ \ x + 2y = 8 \quad \rightarrow (mult\ by - 4) \rightarrow\ (-4 * x) + (-4 * 2y) = (-4 * 8) \quad \rightarrow\ \boldsymbol{-4x - 8x = -32}$

- Add equations to eliminate "*x*"

$Eqn\ 1\text{:}\qquad 4x - 5y = 6$
$\underline{Eqn\ 2\text{:}\ \ -4x - 8x = -32}$
$\qquad\qquad 0x - 13y = -26$

- And solve for "*y*"

$y = \dfrac{-26}{-13} \quad \rightarrow \quad \boldsymbol{y = 2}$

- Substitute "*y*" back into original equation and solve for "*x*"

$4x - 5 * 2 = 6 \ \ \rightarrow \ \ 4x - 10 = 6 \ \ \rightarrow \ \ 4x = 16 \ \ \rightarrow\ x = \dfrac{16}{4} \ \ \rightarrow \ \ \boldsymbol{x = 4}$

- Check the solution:

$(4 * 4) - (5 * 2) = 6\ \ \surd$
$4 + (2 * 2) = 8\ \ \surd$

1) Using the elimination method, determine the solution for the following of systems of equations. Check your answers.
   a) $2a - 4 = b$
   $a + 2b = 7$

   b) $4x + 6 = y$
   $3y + x = 5$

c) $8 = 3r - s$
$r + 4s = 7$

d) $3I_1 + 2I_2 = 2$
$2I_1 + 5I_2 = 2$

Problem Situation 5.6 – Determinants and Cramer's Rule

Yet another method for solving a system of equations is called **Cramer's Rule**. Cramer's rule is a *formula* for solving systems of equations in *matrix form* by using *determinants*.

A **Matrix** is a *rectangular* array of numbers arranged in *rows* and *columns*. Some examples are:

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \qquad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \qquad \begin{vmatrix} a_1 \\ a_2 \\ a_3 \end{vmatrix} \qquad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Size: 2 x 2   2 x 3   3 x 1   3 x 3

A **determinant** is a way to characterize a matrix by doing a mathematical operation, essentially cross-multiplying elements and adding or subtracting them. Determinants are very useful and but can only be defined with a *square* matrix, that is, an *equal* number of row and column elements. *Which of the matrixes above are square?*

To find the determinant of a square matrix, it is defined:

For a 2 x 2 matrix: $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = (a_1 * b_2) - (a_2 * b_1)$

For a 3 x 3 matrix: $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix} = (a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3) - (a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1)$

EXAMPLE:

$$\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = (1 * 4) - (2 * 3) = 4 - 6 = -2$$

1) Find the determinant for the following matrices:

a)
$$\begin{vmatrix} -.24 & 1.54 \\ 2 & -6.2 \end{vmatrix}$$

b)
$$\begin{vmatrix} 1.4 & 2.6 \\ -8.9 & 5.4 \end{vmatrix}$$

c)
$$\begin{vmatrix} -6.1 & 3.1 \\ -3.4 & 16 \end{vmatrix}$$

d)
$$\begin{vmatrix} 2 & 1 & -3 \\ 7 & 5 & 2 \\ -1 & 4 & 3 \end{vmatrix}$$

Cramer's Rule

$a_1x + b_1y = k_1$
$a_2x + b_2y = k_2$

$$x = \frac{\begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & k \\ a_2 & k_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$2x - y = -5$
$3x + 2y = 3$

$$x = \frac{\begin{vmatrix} -5 & -1 \\ 3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}} = \frac{(-5*2)-(3*-1)}{(2*2)-(3*-1)} = \frac{-10+3}{4+3} = \frac{-7}{7} = -1$$

$$y = \frac{\begin{vmatrix} 2 & -5 \\ 3 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}} = \frac{(2*3)-(3*-5)}{7} = \frac{6+15}{7} = \frac{21}{7} = 3$$

*Check: 2 * -1 – 3 = -5 and 3*-1 + 2*3 = 3*

2) Using Cramer's Rule calculate the solution for the following systems of equations.

a)
$6 = 1.5x - y$
$4.5x + y = -6$

b)
$2.3b = a - 2$
$a + b = 2$

c)

$3.4y = 3.8x - 4$
$6y + 4.9x = 6$

d)

$y = 4x - 6$
$6x - 2y = 7$

3) Using Cramer's rule, solve for the currents in our **initial** circuit.

a)

$3I_1 + 2I_2 = 2$
$2I_1 + 5I_2 = 2$

b) Use your calculator's equation solver to solve the same circuit. *Is it the same result? How much harder or easier was it?*