COURSE COMPETENCIES

1. Solve linear equations with one unknown
    - You solve linear equations for one dependent variable
    - You manipulate linear equations to isolate selected variables.
    - You translate real problems into mathematical equations.
    - You apply the mathematics to determine the components, quantities, and properties of DC resistive electrical circuits.
    - Calculate power in DC resistive circuits
    - Analyze series DC resistive circuits
    - Analyze parallel DC resistive circuits
    - Analyze parallel/series DC resistive circuits

BACKGROUND

Kirchhoff's Voltage and Current Laws are fundamental to good electronic design and trouble shooting. The students should be able to proficiently apply Ohm's Law and the Power Rule.

EXPLICIT CONNECTIONS

It is important that each person understands how to apply simple mathematics to complicated circuits. The students should be fluent in applying Kirchhoff's Laws after this module.

NOTES TO SELF

- Encourage each individual to accurately sketch the circuits.
- Have groups sketch and defend their circuit diagrams on the white board.
- Strongly encourage students to make predictions so they continue to build their confidence.

Duration Minutes	Lesson	Suggested Structure
10	Lecture: Review Linear Equations	Cohort
20	Problem Set 4.1: Crazy Taxi	Group
20	Lecture: Linear graphs	Cohort
20	Problem Set 4.2: Functions and Graphing	Group
10	Blackboard: Practice Set 1 - Linear Equations	Individual
15	Blackboard: Practice Set 2 - Linear Graphs	Individual
25	Lecture: Kirchhoff's Voltage Law	Cohort
20	Problem Set 4.3: Kirchhoff's Voltage Law	Group
20	Lecture: Kirchhoff's Current Law	Cohort
30	Problem Set 4.4: Kirchhoff's Current Law	Group
15	Blackboard: Practice Set 3 - Kirchhoff's Laws	Individual
20	Problem Set 4.5: Kicker Circuit	Group
20	Problem Set 4.6: Circuit Analysis	Group
20	Problem Set 4.7: Circuit Puzzles	Group
20	Quiz	Cohort

Lesson	***Objectives***	***Material***
4.1	Linear functions and Graphing	Crazy Taxi
4.2	Linear functions and Graphing	Functions and Graphing
4.3	DC Circuits: Kirchhoff's Voltage Law	Kirchhoff's Voltage Law (KVL)
4.4	DC Circuits: Kirchhoff's Current Law	Kirchhoff's Current Law (KCL)
4.5	DC Circuits: Analysis 1	Kicker Circuit
4.6	DC Circuits: Analysis 2	Circuit Analysis
4.7	DC Circuits: Analysis 3	Circuit Puzzles

Prerequisite Assumptions
Before beginning the lesson, students should understand,

- DC Resistive Circuit equivalent modeling,
- DC Series and parallel connected resistor equivalence,
- Combined DC Parallel and Series connected Resistors
- Application of Ohm's Law and the Power Rule

Specific Objectives
By the end of this lesson, you should be able to;

- ✓ Solve a linear function
- ✓ Understand the slope-intercept form of a linear function
- ✓ Graph a linear function
- ✓ Apply the voltage and current division rules in DC resistive circuit analysis
- ✓ Apply Kirchhoff's Voltage Laws when conducting DC resistive circuit analysis
- ✓ Apply Kirchhoff's Current Laws when conducting DC resistive circuit analysis
- ✓ Apply modeling techniques to analyze complicated DC resistive circuits.

Problem Situation 4.1 – Crazy Taxi

https://tapintoteenminds.com/3act-math/crazy-taxi/

1) What do you think the fare would be for the 30 km traveled?

I would guess about $15.

2) Can you determine the fare for the 30 km traveled?

I need to find the fare for 30 km. The charge is $5 to start and then an additional $0.50 /km. So this would be $5 + $0.50*30 = $20

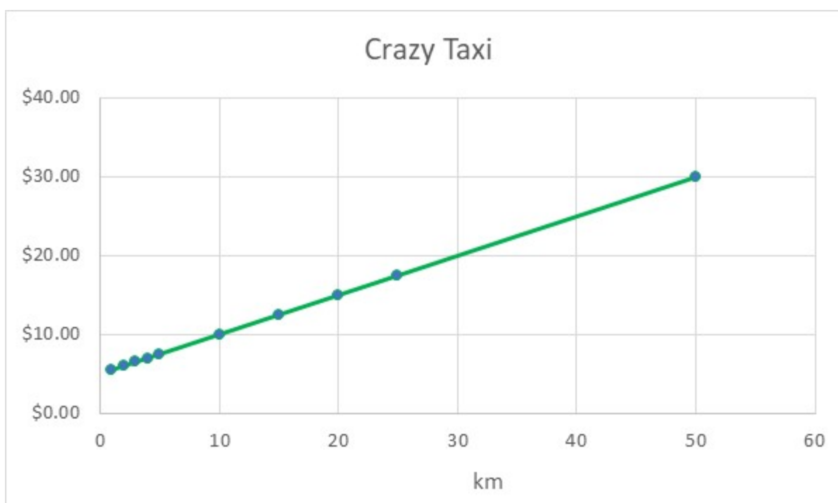
3) Create an equation to model the cost *vs*. km. Can you tell what 'kind' of equation it is?

I need to create and equation so I will first identify the dependent and independent variables that I know. C = total fare and is dependent on d = distance traveled in km.

$C = \$5 + (\$0.50 * d)$ It is a linear equation.

4) Create a graph to show km travelled vs cost. Make sure to label the axis. What is the shape of the curve? Straight line.

Cost ($)	Distance (km)
$5.50	1
$6.00	2
$6.50	3
$7.00	4
$7.50	5
$10.00	10
$17.50	15
$20.00	20
$27.50	25
$30.00	50



5) From the graph, can you determine how far the person could get if they had $25? Can you also calculate the distance?

From the graph, for $25 it looks like it would be about 40 km. For the calculation:

$\$25 = \$5 + (\$0.50 * d) \rightarrow \$25 - \$5 = (\$0.50 * d) \rightarrow \frac{\$20}{\$0.50} = d = 40\ km$

Problem Situation 4.2 – Functions and Graphing

A **function** is the mathematical relationship in which **each** *input* has a **single** *output*. It relates the input (*independent variable*) to the output (*dependent variable*). Typically, this is written as *f(x)* where x is the input to the function *f*. This is read as "f of x". Often *f(x)* will be given another variable name, such as "y", to simplify the amount of writing. This would be written y = *f(x).*

Graphing is very difficult for some students. Understanding the function and graph are worth spending some extra time in the classroom. Data sheets with graphs are frequently used in this field.

Example:
You are having a meeting and want to get cookies for everyone. You order two cookies for each person and then one spare cookie for sampling.

I will use *x* as the "number of people attending the meeting"

f(x) = 2*x* + 1 (2 times *x* plus 1)

Evaluate the function when 2 people are attending: x = 2

f(2) = 2 * 2 + 1

f(2) = 5 thus, for two people you would need 5 cookies

Evaluate the function when 14 people are attending: x = 14

f(14) = 2 * 14 + 1

f(14) = 29 For 14 people you would need 29 cookies.

1) Evaluate the following function when m = 3.5

$$f(m) = \frac{4}{m} * 1.2$$

$$f(3.5) = \frac{4}{3.5} * 1.2 = 1.37$$

2) Evaluate the following function when c = -4.2

$$f(c) = \frac{3}{4}c - 0.9$$

$$f(-4.2) = \frac{3}{4} * -4.2 - 0.9 = -4.05$$

3) Evaluate the following function when *x* = 7.98

$$f(x) = 2.7x + 14$$

$$f(7.98) = 2.7 * 7.98 + 14 = 35.6$$

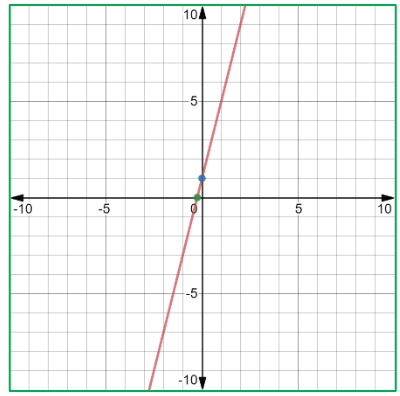
4) Evaluate the following function when *x* = 12

$$f(x) = -3.1x - 1$$

$$f(12) = -3.1 * 12 - 1.2 = -38.4$$

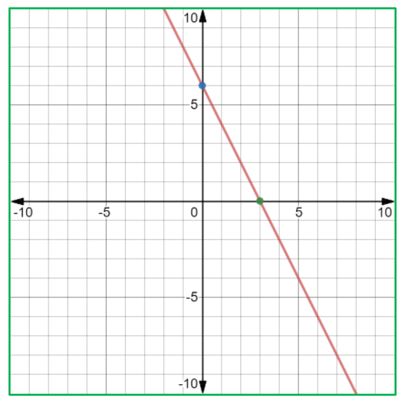
5) Graph the following linear function by determining *at least* two points on the line. Label everything.
$f(x) = 4x + 1$

x	f(x) = y
0	1
-0.25	0



6) Graph the following linear function by determining at least two points on the line. Label everything.
$f(x) = 6 - 2x$

x	f(x) = y
0	6
3	0



A standard description for a linear function is the slope intercept form: $\boldsymbol{y = mx + b}$
where m = the slope or the rise/run and b = y-intercept or where the graph crosses the y-axis

7) Identify the slope and y-intercept for the equations that you just graphed.
$f(x) = y = 4x + 1$

Slope = ____4__________

y-intercept = ____1______

8) $y = 6 - 2x$

Slope = _______-2_________

y-intercept = ____6______

It is well worth spending extra time understanding the standard form of a line, slope and y-intercept.

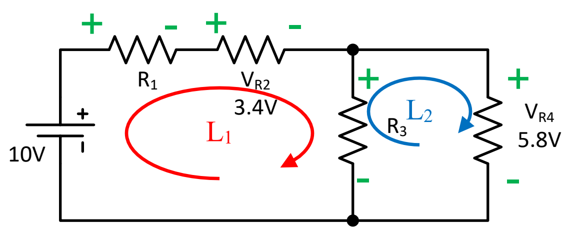
Problem Situation 4.3 – Kirchhoff's Voltage Law (KVL)

Gustav Kirchhoff was a German physicist that did research in the field of electronics in the mid-1800s. He discovered fundamental theories about electronics. The first theory we will study is ***Kirchhoff's Voltage Law***, which states that the voltages (*potential*) in any CLOSED circuit path (a *loop*) always sums to zero. You only get the voltage out that goes in, or all of the voltage is distributed along the loop such that all the voltages cancels out. Voltage potential cannot be lost or gained within a circuit.

Kirchhoff's Voltage Law: In a closed loop, start at ground, the zero-potential point (though you can start anywhere), and trace a closed loop while summing the voltages (– to + is positive and + to – is a negative) you come across until the trace returns to ground (where you started) and you have completed the closed loop. The sum of the voltages of every loop will always equal zero.

Have each group draw their schematic on the white board and explain their decision making.

1) Analyze the following circuit using Kirchhoff's Voltage Law to determine $\mathbf{V_{R1}}$ and $\mathbf{V_{R3}}$.



There are 2 closed loops. I will sum for the voltages in both loops starting at the supply;

$$L_1: 0 = 10V - V_{R1} - V_{R2} - V_{R3}$$
$$0 = 10V - V_{R1} - 3.4V - V_{R3}$$
$$6.6V = V_{R1} - V_{R3}$$

I have 2 independent variables, but I know one of them, so I can solve for the other;

$$L_2: 0 = V_{R3} - V_{R4}$$
$$0 = V_{R3} - 5.8V$$
$$V_{R3} = 5.8V$$

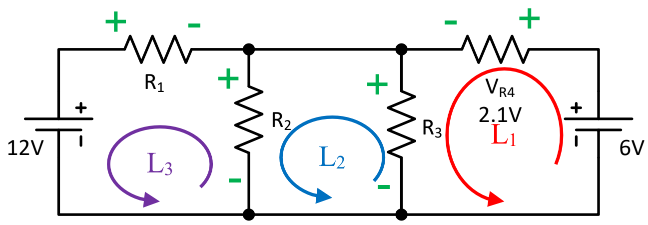
Now I can determine V_{R1} by another substitution with the value of V_{R3};

$$6.6V = V_{R1} - 5.8V$$
$$V_{R1} = 0.8V$$

V_{R1} = ____$0.8V$________

V_{R3} = ______$5.8V$________

2) Analyze the following circuit using Kirchhoff's Voltage Law to determine $\mathbf{V_{R1}}$, $\mathbf{V_{R2}}$ and $\mathbf{V_{R3}}$.



There are three closed loops in this circuit. I need to sum up the closed loops and look for each component to be in at least one loop. L_1 has only one unknown so I will start with that loop at the supply with 'into' a – negative sign being positive and 'into' a + sign being negative.

$$L_1: 0 = 6V - V_{R4} - V_{R3}$$
$$0 = 6V - 2.1V - V_{R3}$$
$$3.9V = V_{R3}$$
$$L_2: 0 = V_{R3} - V_{R2}$$
$$0 = 3.9V - V_{R2}$$
$$3.9V = V_{R2}$$
$$L_3: 0 = V_{R2} + V_{R1} - 12V$$
$$0 = 3.9V + V_{R1} - 12V$$
$$8.1V = V_{R1}$$

V_{R1} = ___8.1V_______

V_{R2} = ___3.9V_________

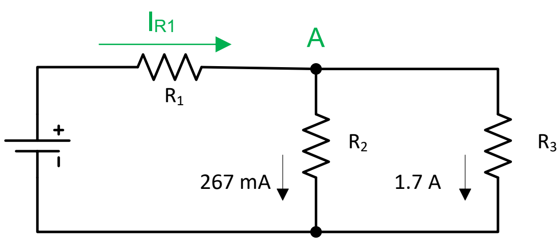
V_{R3} = ___3.9V_________

Problem Situation 4.4 – Kirchhoff's Current Law (KCL)

Gustav Kirchhoff's second law is about the current in a circuit. ***Kirchhoff's Current Law*** states the current at a node (where circuit loops connect) must always sum to zero. Like water pipes coming together, all the current entering the connection point (*node*) must exit. Current will never disappear or grow in a circuit. All the current entering a node must leave that node. It is often helpful to label each node with a number in order to keep track of things. Ground is often labeled "0". KCL is obvious when you only have two paths (*branches*) because you only have a single "in" and a single "out" but it is most useful when you have three or more branches.

Have each group draw their schematic on the white board to explain their decision making process.

1) Analyze the following circuit using Kirchhoff's Current Law to determine $\mathbf{I_{R1}}$.



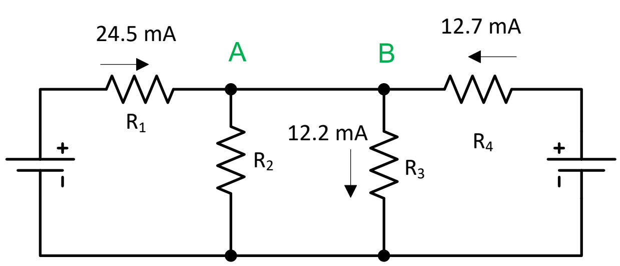
KCL states that the current into any node must sum to zero. I am going first label the node that I will apply KCL to as node A, with 'into' being positive and 'out of' being negative:

$0 = I_{R1} - I_{R2} - I_{R3}$ (I_{R1} *is entering the node,* I_{R2} *and* I_{R3} *are leaving the node.*

$I_{R1} = 0.267A + 1.7A = 1.97A$

I_{R1} = ___$1.97A$_________________

2) Analyze the following circuit using Kirchhoff's Current Law to determine $\mathbf{I_{R2}}$.



KCL states that the current into any node must sum to zero. I am going first label the nodes to which I will apply KCL, node A and B.
I will start with node B because I know the current in 2 of the 3 branches:

$Node\ B: 0 = I_{R4} - I_{R3} - I_A$

$I_A = 12.7mA - 12.2mA = 0.5mA$

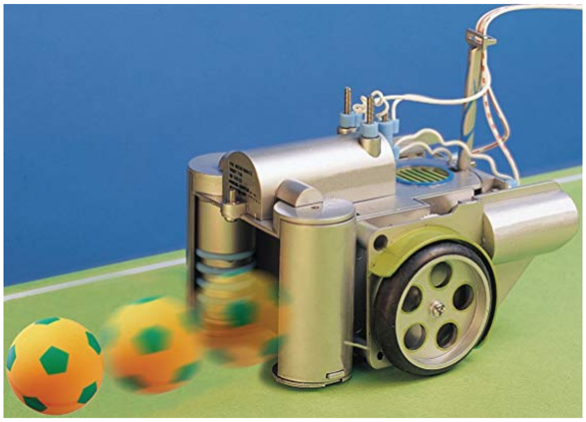
$Node\ A: 0 = I_A + I_{R1} - I_{R2}$

$I_{R2} = 0.5mA + 24.5mA = 25mA$

I_{R2} = __25 mA_________________

Problem Situation 4.5 – Kicker circuit.

The autonomous robot below "sees" a ball and "kicks" it.
The ball activates a switch that energizes the kicker motor (resistive load).
Assume that there are only 20 Ω resistors available to use in the circuit.



1) What requirements do you "know" and what must you assume in order to design a circuit for this application?

System Requirements: Supply, motor load resistance, Current limiting to not burn up motor, Switch.
Design Requirements: Supply, $V_S = 16V$, *required motor voltage,* $V_L = 12V$,
measured motor resistance, $R_L = 30\Omega$, $I_{Lmax} = 400mA$.
These are reasonable assumptions – answers will vary. You may need to tease these out of the students.

2) Sketch your design and give your justifications.

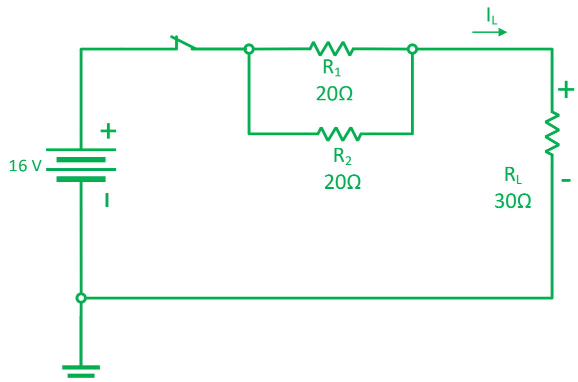
In my design I will need a switch, a voltage source (I am going to assume 16V) and a resistor to model the motor load. Using Ohm's Law I determine the current the motor (R_L) will draw from the supply by itself.

$$I_T = \frac{V_S}{R_L} = \frac{16V}{30\Omega} = 0.533\ A = 533\ mA$$

This is a lot of current, over a half an amp, much more than the 400 mA max. I will need to limit the current so the motor doesn't burn up. In order to meet the design parameters of 400 mA, the **total** resistance needs to be higher.

$$R_T = \frac{V_S}{I_L} = \frac{16V}{0.4A} = 40\Omega$$

I need to have 10Ω added in series with the R_L = 30 Ω to limit the current. I only have 20Ω resistors so two in parallel would give me the required total resistance.



3) How would you "validate" your design to verify that it satisfies the requirements and design parameters both as given and assumed?

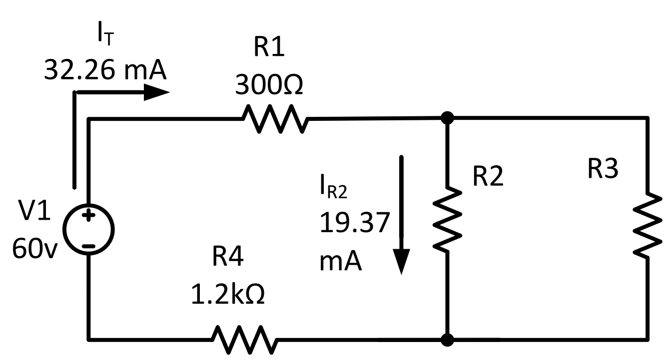
The requirements and design parameters are met because when the "sensor" closes the switch, the total current is 400 mA and this creates a voltage potential of 12V across the motor to run it when the switch is closed.

4) Does your design satisfy the "requirements" as specified for this circuit? Yes.

Problem Situation 4.6 - Circuit Analysis

1) Analyze the following circuit to determine the missing information in the table.

R	Value (Ω)	Current (mA)	Voltage (V)	Power (mW)
R1	300	32.26	9.678	312
R2	600	19.37	11.62	225
R3	900	12.89	11.62	150
R4	1200	32.26	38.7	1,250

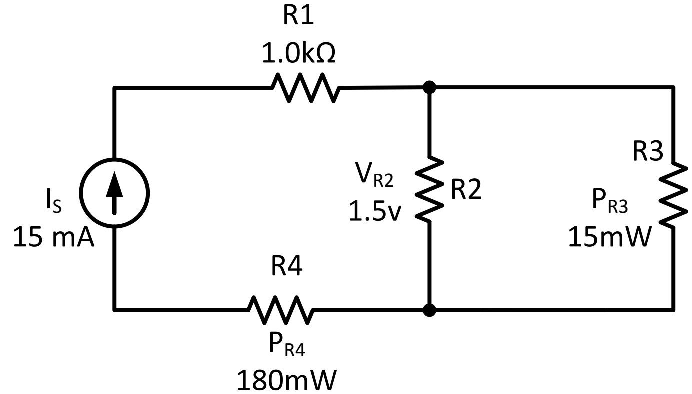


Start by first determining which components you have at least 2 pieces of data. With 2 pieces of data and Ohm's Law you can complete the analysis. Then use KCL and KVL to complete the analysis.

1) R_1 – voltage and power
2) R_4 - current is the same as R_1 so find voltage and power
3) R_2 – use KVL to determine voltage and then value and power
4) R_3 – voltage = voltage as R_2, KCL to determine current and then find value and power

2) Analyze the following circuit to determine the missing information in the table.

R	Value (Ω)	Current (mA)	Voltage (V)	Power (mW)
R1	1000	15	15	225
R2	300	5	1.5	7.5
R3	150	10	1.5	15
R4	800	15	12	180

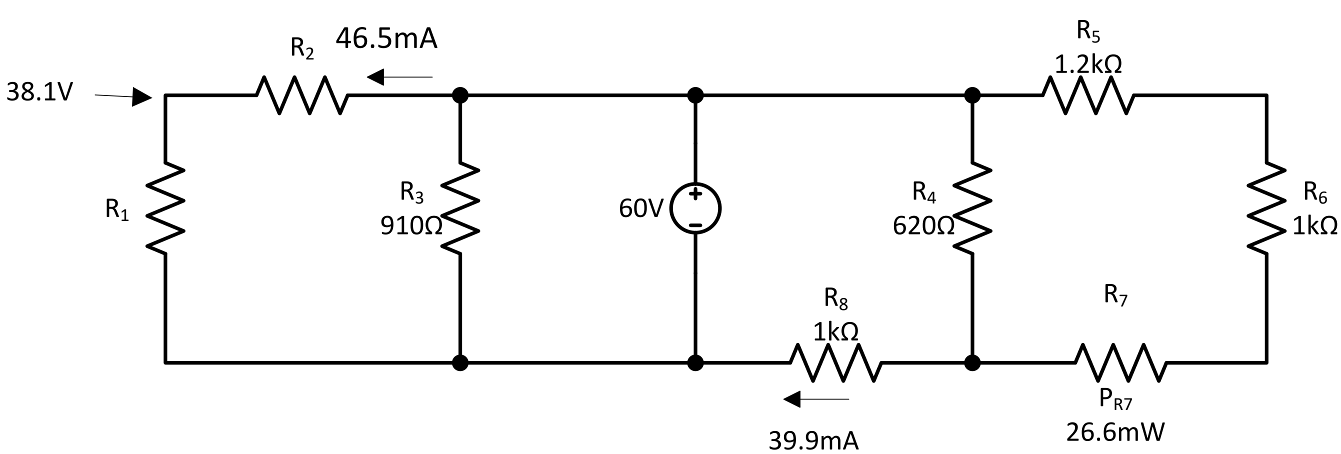


Start by first determining which components you have at least 2 pieces of data. With 2 pieces of data and Ohm's Law you can complete the analysis. Then use KCL and KVL to complete the analysis.

1) R_1 – voltage and power
2) R_4 - current is the same as R_1 so find voltage and value
3) R_2 – use KCL to determine current (current divider) and then value and power
4) R_3 – voltage = voltage as R_2, KCL then find value and current

Problem Situation 4.7 – Circuit Puzzles

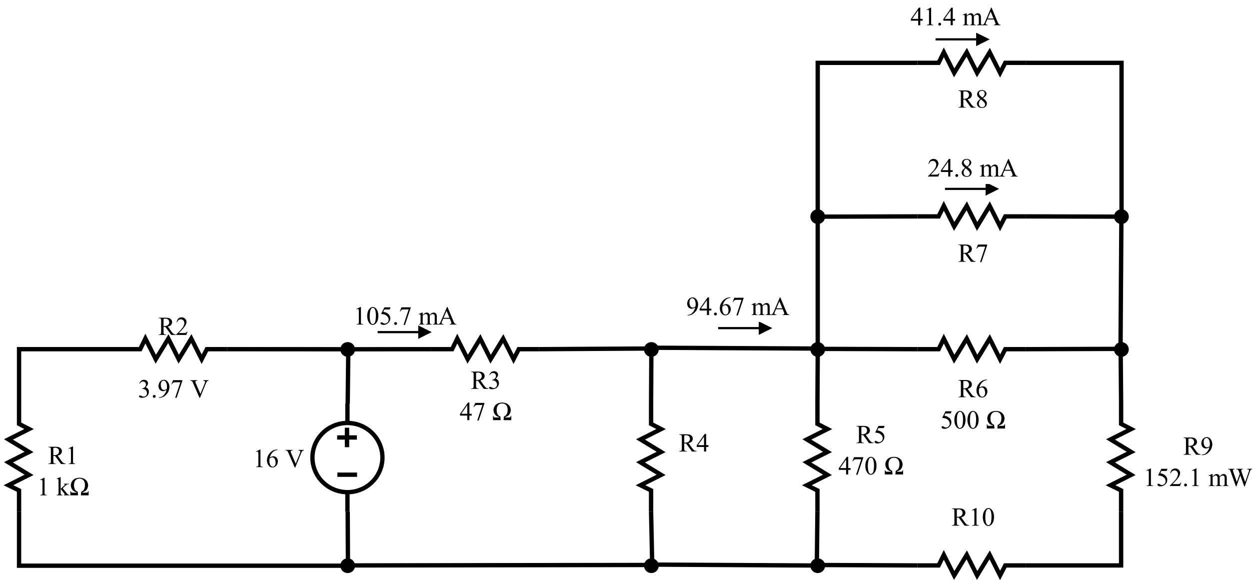
1) Determine the information on the table for the following circuits. Watch your units.



Component	Resistance (Ω)	Current (mA)	Voltage (V)	Power (W)
R_1	820	46.5	38.1	1.77
R_2	470	46.5	21.9	1.02
R_3	910	65.9	60	3.96
R_4	620	32.4	20.1	0.0652
R_5	1200	7.5	9	0.0675
R_6	1000	7.5	7.5	0.0562
R_7	470	7.5	3.5	0.0266
R_8	1000	39.9	39.9	1.59

Start by determining for which components you have at least 2 pieces of data. With 2 pieces of data and Ohm's Law you can solve for an unknown. Then use KCL and KVL to complete the analysis.

1) R_8 – know value and current → calculate voltage and power
2) R_3 - know value and is in parallel with the voltage source → calculate current and power
3) R_1 – know voltage and has the same current as R_2 (in series) → calculate value and power
4) R_2 – I know V_{R3} and V_{R4} so use KVL to determine the voltage, then calculate value and power
5) R_4 – I know the source voltage and V_{R8} so I can use KVL to determine the voltage, then I can determine current and power
6) R_5, R_6 and R_7 are in series so they have the same current. KCL tells me that this current = I_{R8}-I_{R4} so then I have enough information to complete the analysis.



Start by determining for which components you have at least 2 pieces of data. With 2 pieces of data and Ohm's Law you can solve for an unknown. Then use KCL and KVL to complete the analysis.

1) R_3 – know value and current → calculate voltage and power
2) R_1 – use KVL because I know the source voltage and V_{R2} → calculate current and power.
3) R_2 – has the same current as R_1 because they are in series → calculate value and power
4) R_4 – use KCL because I know two of the three branch currents. Use KVL because I know the source voltage and V_{R3} → calculate value and power.
5) R_5 – the voltage is the same as V_{R4}, use KVL → calculate the current and power.
6) R_6 – I know 4 of the 5 currents at that node, use KCL → calculate the voltage and power.
7) R_7 and R_8 – voltage is the same as R_6 because they are in parallel → calculate value and power.
8) R_9 – the current must equal 94.67 mA minus I_{R5} → calculate value and voltage.
9) R_{10} – I_{R10} is the same as I_{R9}, use KVL to determine the voltage → calculate value and power.

Component	Value (Ω)	Current (mA)	Voltage (V)	Power (mW)
R1	1000	12.03	12.03	145
R2	330	12.03	3.97	47.8
R3	47	105.7	4.97	525
R4	1000	11.03	11.03	122
R5	470	23.47	11.03	249
R6	500	5	2.5	12.5
R7	100	24.8	2.5	62
R8	60	41.4	2.5	104
R9	30	71.2	2.14	152.1
R10	90	71.2	6.39	454