

College Algebra Supplemental Material for Biology Majors



This manual is the result of collaborative efforts between faculty from the Dept. of Mathematics and Life Sciences at Los Angeles Mission College and Mathematics faculty at University of California, Los Angeles

This manual is best used in conjunction with Academic Excellence Workshop manual. Website: <https://bit.ly/2QCe0ld>

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WHERE DISCOVERIES BEGIN

College Algebra Academic Success Workshops

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Content Worksheets

Factoring Review

One of the challenges students face in College Algebra is the difficulty to recognize complicated looking problems as needing the same skills as those in earlier classes. To be able to do that, you must first make sure that you understand the reasons behind all the choices in methods we use. Here are some exercises to practice this:

1. Factor $x^5y^5 + x^7y$

Why did you choose the method that you used? Think about all the steps that you took and choices that you made in the factoring this problem. Some of these may have been completely subconscious. Try to apply those same steps to the following problems.

a) Factor $x^{-4}y^{-3} + x^{-5}y^{-1}$

b) Factor $x^{\frac{1}{4}}y^{-\frac{1}{2}} + x^{\frac{1}{3}}y^{\frac{1}{2}}$

c) Factor $(x+2)^{\frac{1}{5}}(y-1)^{-\frac{1}{4}} + (x+2)^{\frac{1}{2}}(y-1)^{\frac{3}{4}}$

2. Factor both $x^2 - 64$ and $x^3 - 64$

Now once again think about all the necessary steps and the choices you had to make. And try these problems:

a) Factor $(x+5)^2 - 64$

b) Factor $x^6 - 8$

c) Factor $x^6 - 4$

d) Factor $x^6 - 64$

3. Factor $x^2 - 2x - 24$

Once you have thought about how you recognized the process needed for these problems, see if you would have recognized that the same process would have been needed for the following, and why?

a) Factor $(x+2y)^2 - 3(x+2y) - 10$

b) Factor $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 48$

4. Simplify : $\frac{x^{\frac{4}{3}} + x^{\frac{2}{3}} - 12}{x^{\frac{5}{3}} + 4x}$

Modeling with Linear Equations

For textbook reference you can use the free opnestax Precalculus text: <https://openstax.org/details/books/prec calculus Sections 2.1-2.3>

A biologist is studying the effects of pollutants such as mercury in stormwater run-off on a local species of frogs. She notices that the relationship between the number of frogs and the amount of mercury present in the water can be modeled linearly. Each sampling site is 1 square kilometer area. In the first site, the water has 0.002 mg of mercury per liter, and there are 5200 frogs present. In the second sampling site further up the river, the water has 0.003 mg of mercury per liter, and there are 4600 frogs presents.

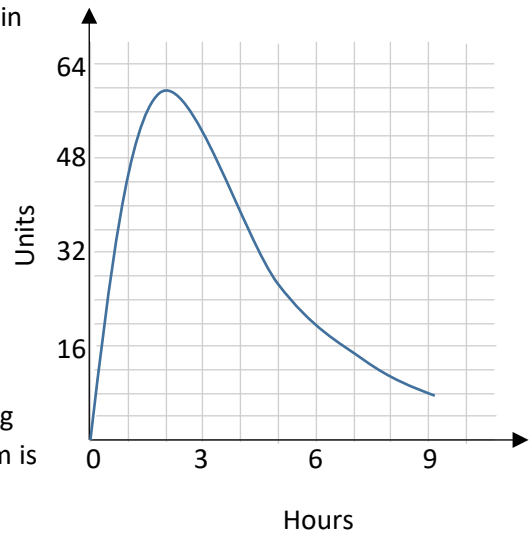
- Write a Linear model that can predict the number of Frogs per square kilometer, F , based on the amount of Mercury per Liter of Water, M .
- Graph the line (Make sure that you only graph the appropriate of the domain and range).
- Find the slope. What does this number tell us about the frogs?
- Find the x-intercept. What does this number tell us about the frogs?
- Find the y-intercept. What does this number tell us about the frogs?
- If there are less than 7000 frogs in the one square kilometer area, the mosquito population will grow unchecked. Mosquitoes carry the plasmodium parasite, which causes malaria in infected humans. What would be the maximum allowable amount of mercury in one liter of water, if we want to prevent the spread of malaria?

2. Crickets chirp by vibrating their wings. Since Crickets are ectotherms (cold-blooded) the rate of their physiological processes and their overall metabolism are influenced by temperature. The relation between the number chirps per second, C , and the temperature in degrees of Fahrenheit, F , can be modeled using a linear equation. Given that crickets chirp 20 times per second when the temperature is 88.6° and 16 times per second when the temperature is 80.6° .

- Write a linear equation for calculating the number of chirps based on the temperature)
- Graph the equation
- Find the slope. What does this number tell us about the crickets?
- Find the x-intercept. What does this number tell us about the crickets?
- Find the y-intercept. What does this number tell us about the crickets?
- At what temperature would the crickets chirp 17 times per second?

Functions and Graphs

1. When a medicine is taken orally, the amount of drug in the bloodstream after t hours is measured and modeled by the function $y = f(t)$, as shown in the graph.
 - a) Find $f(3)$.
 - b) Phrase your answer to part a as a sentence in the context of the medicine.
 - c) When does the level of the drug reach its maximum value and how many units are in the blood stream?
 - d) Write in interval notation, the time interval during which the amount of the drug in the blood stream is increasing.
 - e) When the drug reaches its maximum level in the bloodstream, how many additional hours are required for the level to drop to 20 units?
 - f) Use the graph to describe in words the 9 hour period.



2. Let t represent time measured in hours after 6 AM. The physical activity level of an office worker may be modeled by the following function:

$$f(t) = \begin{cases} t^2 + 5 & \text{if } 0 \leq t \leq 3 \\ 14 & \text{if } 3 < t \leq 9 \\ -2t + 32 & \text{if } 9 < t \leq 16 \end{cases}$$

- a) Graph the function above.
 - b) What are the peak activity hours for this person?
 - c) Is the person more active at 8 AM or 8 PM?
 - d) Find an AM time when the person is equally active as they are at the same PM time.
 - e) In what ways would you expect the activity level of a construction worker to be different? How could you alter the office worker function to reflect this difference?
3. Once a species is introduced in a region, its population's size after t years can be modeled using the function $f(t) = -t^2 + 16t + 40$. The average rate of growth for populations over a period of

time, h , depends on the number of years that has elapsed and can be calculated using the difference quotient $\frac{f(t+h) - f(t)}{h}$.

- a) Use the difference quotient to come up with a general formula for the rate of change.
 - b) Use your answer in part a) to calculate the average rate of change for the population size over a 3 year period starting year 4.
 - c) Using $f(t)$, calculate the population size at year 3 and then again at year 7. What was the average change per year over the 4 years? Does your answer match part b0?
4. When a thermal inversion layer is over a city (as it happens in San Fernando Valley), air pollutants get trapped and cannot rise and therefore must disperse horizontally. Assume that a factory in Sylmar begins emitting a pollutant at 8:00 AM and the pollutant dispersed horizontally over a circular area. If t represents the time, in hours since the factory began emitting pollutants, assume that the radius of the circle of pollutants is $r(t) = 2t$. Recall that the area of a circle is a function of its radius and can be represented using $A(r) = \pi r^2$.
- a) Find $(A \circ r)(t)$
 - b) Interpret $(A \circ r)(t)$ in terms of the pollution.
 - c) What is the area of the circular region covered by the layer at noon?
 - d) Under what conditions would the circular model of pollution dispersion not be realistic? How could you alter the model to make it more realistic in the scenario you identified?

Polynomial Functions

1. Copper in high doses can be lethal to aquatic life. The following table lists the measurements of copper concentration in freshwater, C taken at various distances x kilometers downstream taken by a biologist team studying the pollution impact on freshwater mussels..

x (km)	5	21	37	53	59
C (ppm)	20	13	9	6	5

Statistical software programs can be used to fit various equations to gathered data. Biologists have used such a regression program and have found that the quadratic equation

$C = .00345x^2 - .4933x + 22.23$ fits the data very closely (the data points are very close to the graph of the equation).

- Concentrations above 10 ppm are lethal to mussels. According to the model how far downstream does the toxic water extend?
 - Find the vertex for the quadratic model generated by the computer.
 - Sketch the proposed parabola using its vertex and y-intercept.
 - Discuss the validity of the model for both short and long distances.
 - How would a realistic graph of the water pollution really look?
2. Based on gathered dates, the quadratic function $f(x) = 0.0058x^2 - .038x + 2.660$ models the worldwide atmospheric concentration of carbon dioxide (CO_2) in parts per million (ppm) over the period 1960 – 2013, where $x = 0$ represents the year 1960. If the model continues to apply, answer the following questions:
- What will the concentration of CO_2 be in 2020?
 - What year had the lowest concentration of CO_2 ? What is the lowest concentration of the CO_2 measured?
 - When will the concentration reach 47 ppm?
 - Sketch the proposed parabola using its vertex and y-intercept.
 - Discuss the validity of the model both short term and long term.
 - How are the concerns about the model different in problems 1 and 2?
 - How would a realistic graph of the water pollution really look?
3. As you have noticed in the above problems, quadratic equations are not always the best models for gathered data. To familiarize ourselves with other types of polynomials that can be used we will examine a cubic function $f(x) = 14x^3 + 4x^2 - 8x - 10$
- List all possible rational roots.
 - Test the roots to see if any work.
 - Find all the roots (real or imaginary)
 - Sketch a graph for the above equation using its intercepts and the sign for y value.

Rational Functions

- 1) Ecologists and evolutionary biologists often use the size of the population, N (census population size) to assess population health and to identify processes that shape evolution. For example, genetic drift plays a larger role in smaller populations. Population size, however, may not always be a good measure. For example, a population may consist of 95% sexually immature and 5% sexually mature individuals. In such cases, **effective population size** (N_e) may be computed based on the number of individuals that actually take part in reproduction. Effective population size and census population size may be drastically different when the number of breeding males and breeding females in a population are not close (e.g. polygamous species and social insects). In this case, we can express effective population size using a rational function as,

$$N_e = \frac{4N_f N_m}{N_f + N_m},$$

Where N_f and N_m are the number of breeding females and breeding males, respectively. We will also assume that, $N_f + N_m = N$.

- a) If the number of breeding males and females are equal, how does the effective population size compare to the census size?
 - b) If the effective size of a given population is 1000 and the number of breeding females is a 400, what is the number of breeding males?
 - c) In most insect colonies, the number of breeding females is fixed, while the number of breeding males may fluctuate based on ecological factors. As the number of breeding males increases, the supply of food and competition will eventually limit the effective size of the population. Suppose that an insect colony has 500 breeding females. Find the limit on the effective size of the population as the number of males gets bigger.
 - d) Would this function have any vertical asymptotes? Explain.
- 2) To further examine rational functions, we will use your calculator to study the function's behavior for very large values. For each of the following functions complete the table for each function, and in your words describe what happens to the function.

a) $f(x) = \frac{1}{x}$

x	$f(x)$
10	
100	
1000	
10,000	

b) $f(x) = \frac{5}{x+2}$

x	$f(x)$
10	
100	
1000	
10,000	

c) $f(x) = \frac{5x}{x+2}$

x	$f(x)$
10	
100	
1000	
10,000	

d) $f(x) = \frac{6x-4}{2x+1}$

x	$f(x)$
10	
100	
1000	
10,000	

e) $f(x) = \frac{6x-4}{2x^2+1}$

x	$f(x)$
10	
100	
1000	
10,000	

f) $f(x) = \frac{6x^2-4}{2x+1}$

x	$f(x)$
10	
100	
1000	
10,000	

g) $f(x) = \frac{2x^2-4}{6x^2+1}$

x	$f(x)$
10	
100	
1000	
10,000	

h) Look back at each of the functions and see if you can form a hypothesis on the effects of the exponents, coefficients, and the constant terms on the end behavior of each function.

3) The activity of an immune system invaded by a parasite can be modeled by the function

$$f(n) = \frac{an^2}{b+n^2}, \text{ where } n \text{ measures the number of larvae in a host, } a \text{ is a constant that}$$

depends on the type of parasite and b is a measure of the sensitivity of the immune system. The immune system's activity increases as the number of larvae increase. If for a certain parasite $a = 0.2$ and the host's immune system's sensitivity is given by $b = 17$, find the maximum activity for the host's immune system.

Exponential Functions

1. Exponential functions can be used to model the concentration of a drug in a patient's body. Suppose the concentration of Drug X in a patient's bloodstream is modeled by $C(t) = C_0 e^{-rt}$ where $C(t)$ represents the concentration at time t (in hours), C_0 is the concentration of the drug in the blood immediately after injection, and $r > 0$ is a constant indicating the removal of the drug by the body through metabolism and/or excretion. The rate constant r has units of 1/time (1/hr). It is important to note that this model assumes that the blood concentration of the drug (C_0) peaks immediately when the drug is injected. (Source: *Biology Project, University of Arizona*)
- Suppose that a drug has a removal rate constant of $r = 0.080$ 1/hr and an initial concentration (C_0) of 5.0 mg/L. Find the drug concentration after 4.0 hours to the nearest tenth
 - If the initial concentration of a drug is 10 mg/L, and after 2 hours the concentration reduces to 8.5 mg/L, find the constant r to the nearest hundredth.
 - If for Drug X, $r = 0.20$ 1/hr, how long after injection does the concentration of a Drug X decrease to 35% of its initial level? Round your answer to the nearest tenth.
 - Suppose Drug X is ineffective when the concentration drops below 0.50 mg/L. If $r = 0.14$ 1/hr, what concentration must be initiated if the concentration is to be 0.50 mg/L after 6.0 hours? Round your answer to the nearest tenth.
 - Suppose a drug is administered with an initial concentration of 2.60 mg/L. If $r = 0.120$ for this drug, and the drug is ineffective when the concentration falls below 0.650 mg/L, what is the approximate maximum time a nurse can wait to administer another dose?
 - If $r = 0.22$ 1/hr for a particular drug, how long does it take for the concentration to be half the initial concentration?
 - The data below correspond to a particular drug. Using the data, find the value of the constant r .

Time (hours)	Concentration (mg/L)
2.00	3.63
5.00	2.47

- 3) A fundamental population growth model in ecology is the *logistic model*. This model is more realistic than exponential growth in that logistic growth is not unbounded: the logistic model assumes that in the long run, due to competition for resources, the environment can only support some finite number of animals, called the *carrying capacity*. We can write the logistic model as

$$P(t) = \frac{P_0 \cdot K}{P_0 + (K - P_0) \cdot e^{-rt}}$$

where $P(t)$ is the population size at time t (assume that time is measured in days), P_0 is the initial population size, K is the carrying capacity of the environment, and r is a constant representing the rate of population growth or decay independent of competition for resources. (Source: *Biology Project, University of Arizona*)

- a) Given an initial population of 100 individuals, a carrying capacity of 250 individuals, and $r = 0.4$, compute the expected population size 4 days later.
- b) Repeat the calculation in part a with an initial population of 400. Was there a change?
- c) How long will it take a population of 13 with a carrying capacity of 80 to double given $r = 0.2$?
- d) Let $r = 0.34$, $K = 100$, and $P_0 = 12$. Given these value compute the population size when $t = 5$, $t = 10$, $t = 25$, $t = 100$, and $t = 1000$ days. What do you notice? What do you suggest happens to the population size farther into the future (as $t \rightarrow \infty$) ?

Systems of Equations

1. A chemist has two different alcohol solutions available. One solution contains 5% alcohol and the other 12% alcohol. How much of each should be mixed to obtain 1250 gal of a solution containing 10% alcohol?
2. The following table shows the number of bacteria present, in millions, measured on a surface during 5 hours. Find a function of the form $f(x) = ax^2 + bx + c$ that fits this data. Using this function, what would you predict the number of bacteria present at hour 10?

Hour	Bacteria present (in millions)
0	20
3	29
5	85

3. A zoologist wishes to plan a meal for a new zoo animal around three available foods. The percentage of the daily requirements of proteins, carbohydrates, and iron contained in each ounce of the three foods is summarized in the following table: Determine how many ounces of each food the zoologist should include in the meal to meet exactly the daily requirement of proteins, carbohydrates, and iron (100% of each) for the animal.

	Food I	Food II	Food III
Protein (%)	10	6	8
Carbohydrates (%)	10	12	6
Iron (%)	5	4	12

4. Animals tend to migrate around their habitat based on the availability of resources and migration patterns of other animals. The annual migration paths of different species can be plotted on map grids, and biologists can find equations for the paths. These paths are then monitored to protect various populations from coming across new and more aggressive species or the effects of new human interaction. Suppose that for practical purposes the migration pattern for a population of antelope in an animal sanctuary can be approximately modeled to have equation $x^2 + y^2 = 16$ while the path of lions in the same sanctuary can be approximated by the equation $4x^2 + 25y^2 = 100$.
 - a) Find all the places on the grid where their paths cross. Do you think that the antelope are necessarily in danger? Why or why not?
 - b) To avoid tunneling through a mountain, a new proposed road would cut through the sanctuary. If the road's path in the park has equation $y - x^2 = 3$. Will the road cross the migration path for the Antelope? How about the Lions?

Sequences

1. Frequently the populations of animals grow rapidly at first and then level off because of competition for limited resources. In one study, the population of winter moth was modeled with the sequence given below, where a_n represent the population density per acre during year n :

$$a_1 = 1$$

$$a_n = 2.71a_{n-1} - 0.17(a_{n-1})^2, \text{ for } n \geq 2$$

Write the sequence for $n = 1, 2, 3, \dots, 10$

n										
a_n										

Describe what happens to the population density.

2. If certain bacteria are cultured in a medium with sufficient nutrients, they will double in size and then divide every 30 minutes. Let N_1 be the initial number of bacteria cells, N_2 be the number after 30 minutes, N_3 be the number after 60 minutes, and N_j the number after $30(j-1)$ minute,
- Write N_{j+1} in terms of N_j for $j \geq 1$.
 - If $N_1 = 410$ find the number of bacteria after 3 hours.
 - Describe the growth of these bacteria.
3. If the bacteria are not cultured in a medium with sufficient nutrients, competition will slow the growth. According to Verhulst's model, the number of bacteria N_j after $30(j-1)$ minutes can be modeled using

$$N_{j+1} = \left[\frac{2}{1 + \frac{N_j}{K}} \right] N_j, \text{ where } K \text{ is a constant and } j \geq 1$$

- $N_1 = 410$ and $K = 3000$ calculate the population for the first 10 hours. (Round to the nearest integer).
- Describe the growth of the bacteria with limited nutrients and compare it with the bacteria in problem 3.
- K is called the saturation constant. Why do you think that is the name given to K ? Try the above problems with different values of K .

4. Male honeybees hatch from eggs that have not been fertilized, so a male bee only has one parent (a female). On the other hand, female honeybees hatch from fertilized eggs, so a female has two parents (one male and one female).
- Following the description above, make a diagram showing the number of ancestors for in each generation for a male bee for 5 generations.
 - Write the numbers as a sequence. What kind of a sequence is this?
 - Write a formula for the sequence.
 - Using F_n , M_n and P_n for female, male and total population for each generation, try to find a relation between the current population and the previous generations of males and females.

Test Reviews

Cartesian Coordinate Systems and Functions Review

Graph the equation.

1) $5x + y = -1$

2) $y = x^2 + 2$

3) The points $(-2, -3)$ and $(-2, 6)$ are the endpoints of the diameter of a circle. Find the length of the radius of the circle.

Find the midpoint of the segment having the given endpoints.

4) $(-3, 7)$ and $(-4, 4)$

Find an equation for the circle.

5) Endpoints of a diameter $(3, -4)$, $(3, 4)$

Find the center and radius of the circle.

6) $(x + 6)^2 + (y + 9)^2 = 1$

Evaluate as requested.

7) Given that $f(x) = \frac{x}{11 - x}$, find $f\left(-\frac{2}{5}\right)$.

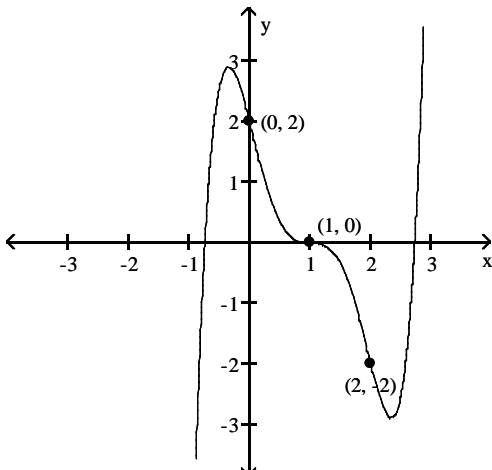
8) Given that $g(x) = 5x^3$, find $g(5 + h)$.

Graph the function.

9) $f(x) = \sqrt{x} + 4$

10) $f(x) = \sqrt{x + 3}$

11) A graph of a function f is shown below. Find $f(0)$.



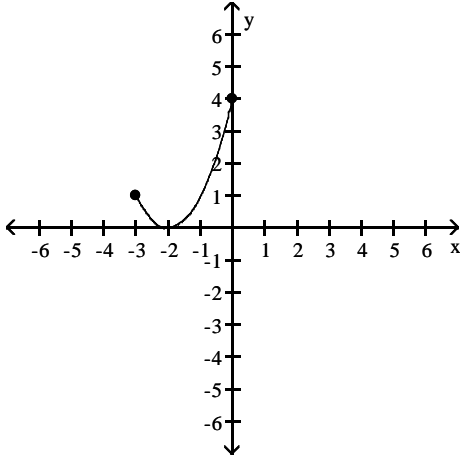
Find the domain of the function.

12) $f(x) = \sqrt{1 - x}$

13) $f(x) = \frac{1}{x^2 + 5x - 14}$

Find the domain and range of the function represented in the graph.

14)



15) Suppose the sales of a particular brand of appliance satisfy the relationship $S(x) = 130x + 5200$, where $S(x)$ represents the number of sales in year x , with $x = 0$ corresponding to 1982. In what year would the sales be 6110?

16) Marty's Tee Shirt & Jacket Company is to produce a new line of jackets with an embroidery of a Great Pyrenees dog on the front. There are fixed costs of \$670 to set up for production, and variable costs of \$44 per jacket. Write an equation that can be used to determine the total cost, $C(x)$, encountered by Marty's Company in producing x jackets.

17) Find a linear function, h , given $h(-8) = -17$ and $h(8) = 15$.

Determine whether the pair of lines is parallel, perpendicular, or neither.

18) $3x - 4y = 6$
 $8x + 6y = 6$

Solve the problem.

19) In triangle ABC , angle A is three times as large as angle C . The measure of angle B is 15° less than that of angle C . Find the measure of the angles.

Solve and write interval notation for the solution set. Then graph the solution set.

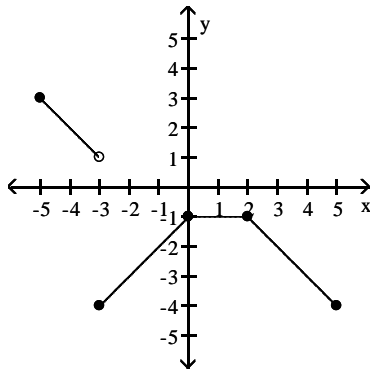
20) $4 < 1 - 4x \leq 12$

Solve and give interval notation for the solution set. Then graph the solution set.

21) $-7x + 1 \geq 15$ or $3x + 3 \geq -9$

Determine the intervals on which the function is increasing, decreasing, and constant.

22)



23) Elissa wants to set up a rectangular dog run in her backyard. She has 36 feet of fencing to work with and wants to use it all. If the dog run is to be x feet long, express the area of the dog run as a function of x .

Solve.

24) From a 15-inch by 15-inch piece of metal, squares are cut out of the four corners so that the sides can then be folded up to make a box. Let x represent the length of the sides of the squares, in inches, that are cut out. Express the volume of the box as a function of x . Graph the function and from the graph determine the value of x , to the nearest tenth of an inch, that will yield the maximum volume.

Graph the function.

$$25) f(x) = \begin{cases} 3 - x, & \text{for } x \leq 2, \\ 1 + 2x, & \text{for } x > 2 \end{cases}$$

Solve.

26) At Allied Electronics, production has begun on the X-15 Computer Chip. The total revenue function is given by $R(x) = 47x - 0.3x^2$ and the total cost function is given by $C(x) = 6x + 13$, where x represents the number of boxes of computer chips produced. The total profit function, $P(x)$, is such that $P(x) = R(x) - C(x)$. Find $P(x)$.

For the function f , construct and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$.

$$27) f(x) = 6x^2 + 3x$$

Find $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$.

$$28) h(x) = \sqrt{\frac{x+2}{x-1}}$$

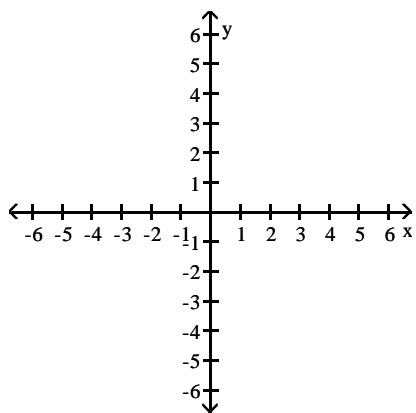
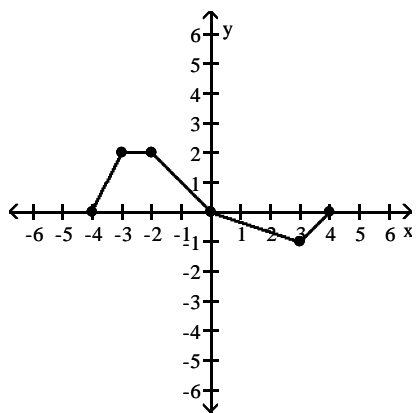
Answer the question.

29) How can the graph of $f(x) = \frac{1}{2}(x+2)^2 - 5$ be obtained from the graph of $y = x^2$?

30) The resistance of a wire varies directly as the length of the wire and inversely as the square of the diameter of the wire. A 20 foot length of wire with a diameter of 0.1 inch has a resistance of 3 ohms. What would the resistance be for a 37 foot length, with diameter 0.01 inch, of the same kind of wire?

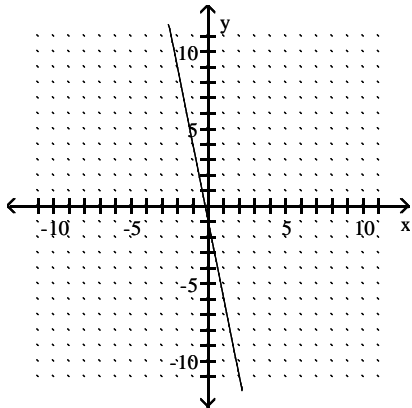
A graph of $y = f(x)$ follows. No formula for f is given. Graph the given equation.

31) $y = f(2x)$

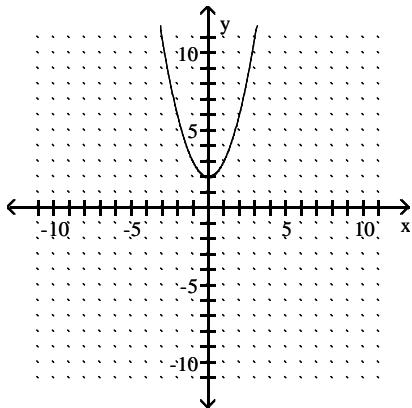


Answer Key

1)



2)



3) 4.5

4) $\left(-\frac{7}{2}, \frac{11}{2}\right)$

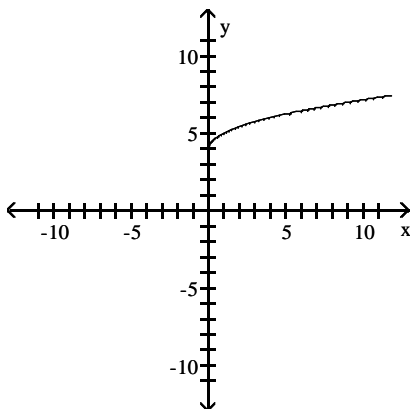
5) $(x - 3)^2 + y^2 = 16$

6) $(-6, -9); 1$

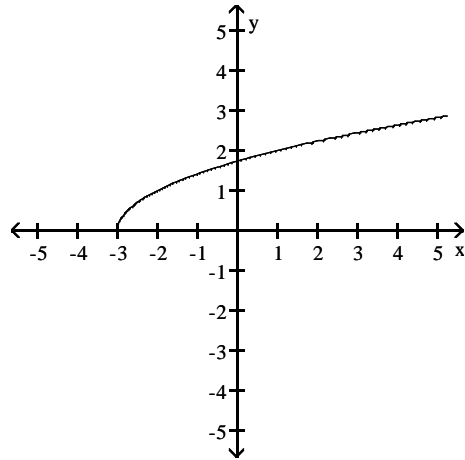
7) $-\frac{2}{57}$

8) $625 + 375h + 75h^2 + 5h^3$

9)



10)



11) 2

12) $\{x \mid x \leq 1\}$, or $(-\infty, 1]$

13) $\{x \mid x \neq -7 \text{ and } x \neq 2\}$, or $(-\infty, -7) \cup (-7, 2) \cup (2, \infty)$

14) Domain: $[-3, 0]$; Range: $[-0, 4]$

15) 1989

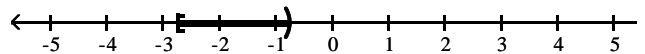
16) $C(x) = 670 + 44x$

17) $h(x) = 2x - 1$

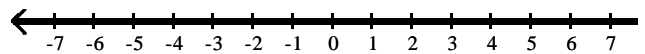
18) Perpendicular

19) $117^\circ, 24^\circ$ and 39°

20) $\left[-\frac{11}{4}, -\frac{3}{4}\right)$



21) $(-\infty, \infty)$

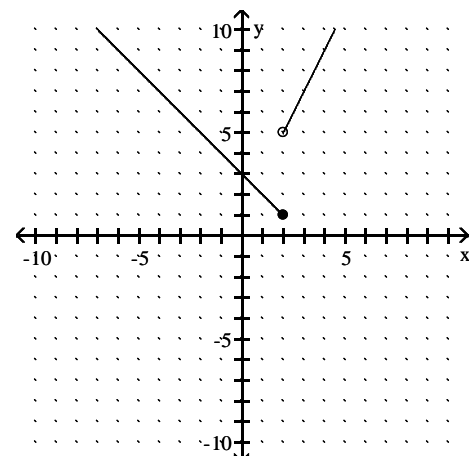


22) Increasing on $(-3, 0)$; Decreasing on $(-5, -3)$ and $(2, 5)$;
Constant on $(0, 2)$

23) $A(x) = 18x - x^2$

24) 2.5 inches

25)



Answer Key

Testname: CARTESIAN COORDIANTE SYSTEMS AND FUNCTIONS

26) $P(x) = -0.3x^2 + 41x - 13$

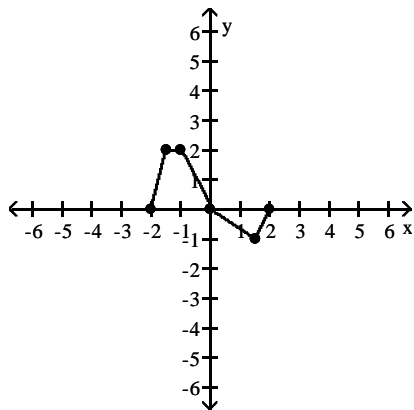
27) $12x + 6h + 3$

28) $f(x) = \sqrt{x}$, $g(x) = \frac{x+2}{x-1}$

29) Shift it horizontally 2 units to the left. Shrink it vertically by a factor of $\frac{1}{2}$. Shift it 5 units down.

30) 555 ohms

31)



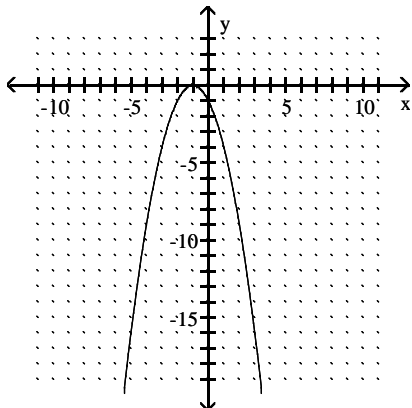
Solving Equations Review

- 1) Express the number in terms of i : $\sqrt{-20}$
- 2) Simplify: $(9 - 5i)(2 + 4i)$
- 3) Simplify: $(7 - \sqrt{-64}) + (4 + \sqrt{-9})$
- 4) Simplify: $(7 - 3i)^2$
- 5) Simplify: $\frac{8 + \sqrt{3}i}{4 - 2i}$
- 6) Simplify: $\frac{i}{5 + i}$
- 7) Simplify: i^{17}
- 8) Simplify: i^{10}
- 9) Solve: $x^2 + 12x + 36 = 17$
- 10) Find the zeros of the function. Give exact answers: $f(x) = x^2 - 5x + 1$
- 11) Solve: $(2p - 4)^2 = 5(2p - 4) + 6$
- 12) Solve: $x^{2/3} - 7x^{1/3} + 10 = 0$
- 13) A grasshopper is perched on a reed 5 inches above the ground. It hops off the reed and lands on the ground about 5.9 inches away. During its hop, its height is given by the equation $h = -0.4x^2 + 1.50x + 5$, where x is the distance in inches from the base of the reed, and h is in inches. How far was the grasshopper from the base of the reed when it was 3.75 inches above the ground? Round to the nearest tenth.
- 14) The area of a square is 81 square centimeters. If the same amount is added to one dimension and removed from the other, the resulting rectangle has an area 9 square centimeters less than the area of the square. How much is added and subtracted?
- 15) Find the vertex of the parabola: $f(x) = -3x^2 + 24x - 51$
- 16) Graph: $f(x) = -x^2 - 2x - 1$
- 17) Find the range of the given function: $f(x) = -2x^2 - 12x - 14$

- 18) Find the intervals on which $f(x)$ is increasing and the intervals where it is decreasing, $f(x) = -3x^2 + 18x + 81$
- 19) A projectile is thrown upward so that its distance above the ground after t seconds is $h(t) = -16t^2 + 420t$. After how many seconds does it reach its maximum height?
- 20) The number of mosquitoes $M(x)$, in millions, in a certain area depends on the June rainfall x , in inches, according to the function $M(x) = 3x - x^2$. What rainfall produces the maximum number of mosquitoes?
- 21) Solve: $\frac{1}{t} + \frac{1}{3t} + \frac{1}{5t} = 9$
- 22) Solve: $\frac{4}{m+3} + \frac{5}{m} = \frac{3m+3}{m^2+3m}$
- 23) Solve: $\frac{1}{x-6} - \frac{1}{x} = \frac{6}{x^2-6x}$
- 24) Solve: $\sqrt[3]{2x-5} + 9 = 5$
- 25) Solve: $\sqrt{3x+14} = x+3$
- 26) Solve: $\sqrt{2x+3} - \sqrt{x+1} = 1$
- 27) Solve: $x^{1/3} = -5$
- 28) Solve $\frac{1}{Q} = \frac{1}{T_1} + \frac{1}{T_2}$, for T_2
- 29) Solve: $Z = A(1+x)^{1/3}$, for x
- 30) Solve: $8 - |4x+3| = 5$
- 31) Solve: $|3x+5| = 6$
- 32) Solve and write interval notation for the solution set: $|7x+5| < 15$
- 33) Solve and write interval notation for the solution set: $|11x+7| < 0$
- 34) Solve and write interval notation for the solution set: $\left| \frac{4x-1}{5} \right| > 4$
- 35) Solve and write interval notation for the solution set: $|3x-7| > -6$

Answer Key

- 1) $2\sqrt{5}i$
- 2) $38 + 26i$
- 3) $11 - 5i$
- 4) $40 - 42i$
- 5) $\frac{32 - 2\sqrt{3}}{20} + \frac{16 + 4\sqrt{3}}{20}i$
- 6) $\frac{1}{26} + \frac{5}{26}i$
- 7) i
- 8) -1
- 9) $-6 + \sqrt{17}, -6 - \sqrt{17}$
- 10) $\frac{5 \pm \sqrt{21}}{2}$
- 11) $\frac{3}{2}, 5$
- 12) $8, 125$
- 13) 4.5 in.
- 14) 3 cm
- 15) $(4, -3)$
- 16)



- 17) $(-\infty, 4]$
- 18) Increasing on $(-\infty, 3)$; decreasing on $(3, -\infty)$
- 19) 13 sec
- 20) 1.5 in.
- 21) $\frac{23}{135}$
- 22) -2
- 23) $\{x \mid x \text{ is a real number and } x \neq 0 \text{ and } x \neq 6\}$
- 24) $-\frac{59}{2}$
- 25) $\frac{-3 + \sqrt{29}}{2}$
- 26) $3, -1$
- 27) -125
- 28) $T_2 = \frac{QT_1}{T_1 - Q}$

$$29) x = \left(\frac{Z}{A}\right)^3 - 1$$

$$30) -\frac{3}{2}, 0$$

$$31) -\frac{11}{3}, \frac{1}{3}$$

$$32) \left[-\frac{20}{7}, \frac{10}{7}\right]$$

33) No solution

$$34) \left(-\infty, -\frac{19}{4}\right) \cup \left(\frac{21}{4}, \infty\right)$$

$$35) (-\infty, \infty)$$

Polynomials and Rational Functions Review

- 1) Is -3 a zero of the function $f(x) = x^4 - 3x^2 - 54$?
- 2) Find the zeros of the polynomial function $f(x) = -6x^2(x - 9)(x + 1)^3$ and state the multiplicity.
- 3) Find the zeros of the polynomial function $f(x) = x^3 + x^2 - 3x - 3$ and state the multiplicity.
- 4) Assume that a person's threshold weight W , defined as the weight above which the risk of death rises dramatically, is given by $W(h) = \left(\frac{h}{12.3}\right)^3$, where W is in pounds and h is the person's height in inches. Find the threshold weight for a person who is 6 ft 1 in. tall. Round your answer to the nearest pound.
- 5) $A(x) = -0.015x^3 + 1.05x$ gives the alcohol level in an average person's bloodstream x hours after drinking 8 oz of 100-proof whiskey. If the level exceeds 1.5 units, a person is legally drunk. Would a person be drunk after 4 hours?
- 6) **Graph the function:** $f(x) = 2x(x + 1)(x - 2)$
- 7) **Graph the function:** $f(x) = -x^4 - 3x^2$
- 8) **Graph the function:** $f(x) = x^4 - 5x^3 + 6x^2$
- 9) **Graph the function:** $f(x) = x^3 + 3x^2 - x - 3$
- 10) **Graph the piecewise function:** $f(x) = \begin{cases} -x + 6, & \text{for } x < -2, \\ 9, & \text{for } -2 \leq x < 0, \\ x^2 - 5, & \text{for } x \geq 0 \end{cases}$
- 11) Divide $x^3 - x^2 + 4$ by $x + 2$ using long division.
- 12) **Divide using synthetic division:** $(2x^4 - x^3 - 15x^2 + 3x) \div (x + 3)$
- 13) Find a polynomial function of degree 3 with $-2, 3, 5$ as zeros.
- 14) Find a polynomial function of degree 3 with $5, 2i, -2i$ as zeros.
- 15) Suppose that a polynomial function of degree 4 with rational coefficients has $6, 4, 3i$ as zeros. Find the other zero.
- 16) **Given that 2 is a zero of the given polynomial, find the other zeros.** $f(x) = x^3 - 4x^2 + 9x - 10$; 2
- 17) **List all possible rational zeros for the polynomial.** $f(x) = -2x^4 + 4x^3 + 3x^2 + 18$

18) Find the rational zeros. $f(x) = x^3 - 8x^2 + 4x + 48$

19) Use Descartes' Rule of Signs to determine the possible number of positive real zeros and the possible number of negative real zeros for the function. $F(x) = 9x^5 - 5x^4 + 3x^3 - 3$

20) State the domain of the rational function. $f(x) = \frac{x - 4}{x^2 + 5x}$

21) Find the vertical asymptote(s) of the graph: $h(x) = \frac{x^2 - 100}{(x - 2)(x + 4)}$

22) Find the horizontal asymptote, if any: $f(x) = \frac{x^2 + 3x + 2}{9 - x^2}$

23) Find the horizontal asymptote, if any: $f(x) = \frac{x^2 + 5x - 7}{x - 7}$

24) Graph: $f(x) = \frac{x - 4}{x + 5}$

25) Graph: $f(x) = \frac{x^2 - 16}{x - 4}$

26) The function $N(t) = \frac{0.6t + 1000}{6t + 5}$, $t \geq 8$ gives the body concentration $N(t)$, in parts per million, of a certain dosage of medication after time t , in hours.

27) Find a rational function that satisfies the given conditions: Vertical asymptotes $x = -3$, $x = 6$; horizontal asymptote $y = 2$; x-intercept $(2, 0)$

28) For the function $g(x) = \frac{x - 1}{x + 8}$, solve $g(x) > 0$.

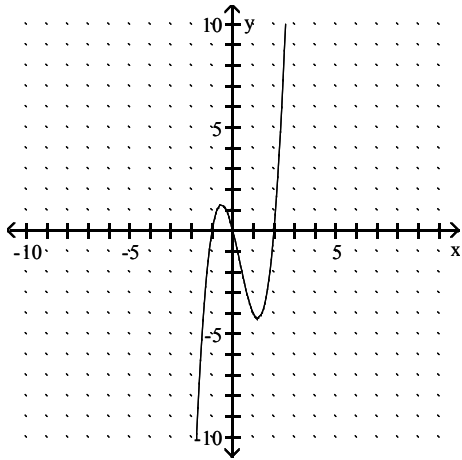
29) Solve. $x^5 - 4x^3 \geq 0$

30) Solve: $\frac{x}{x - 4} < 3$

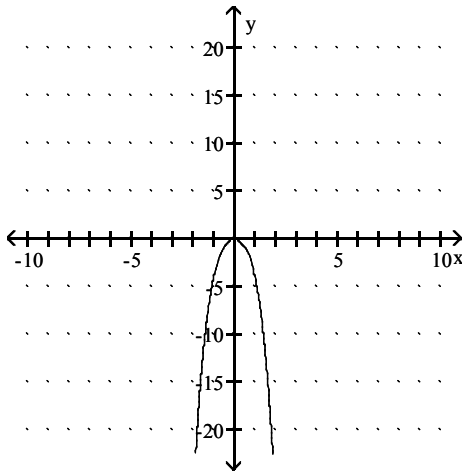
31) Suppose that the temperature T , in degrees Fahrenheit, of a person during an illness is given by the function $T(t) = \frac{4t}{t^2 + 1} + 98.6$, where t is the time, in hours. Find the interval on which the temperature is over 100° .

Answer Key

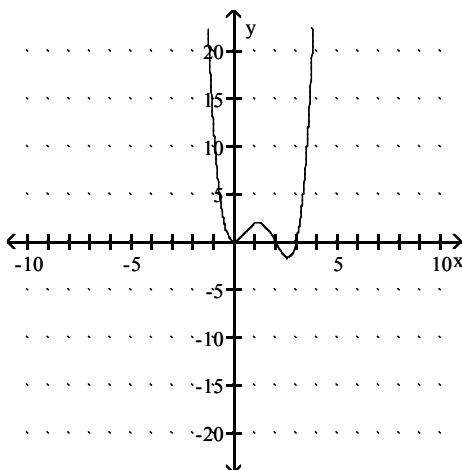
- 1) Yes
- 2) -1, multiplicity 3; 0, multiplicity 2; 9, multiplicity 1
- 3) -1, multiplicity 1; $\sqrt{3}$, multiplicity 1; $-\sqrt{3}$, multiplicity 1
- 4) 209.1 lb
- 5) Yes
- 6)



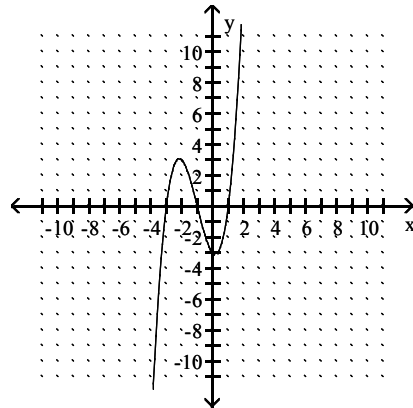
7)



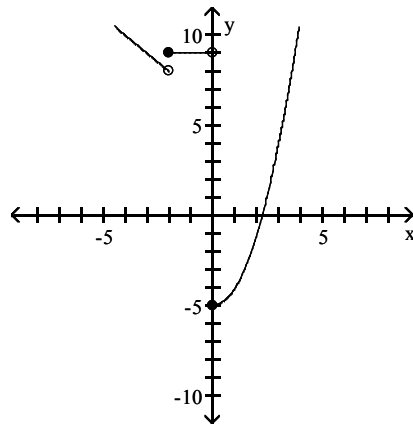
8)



9)



10)

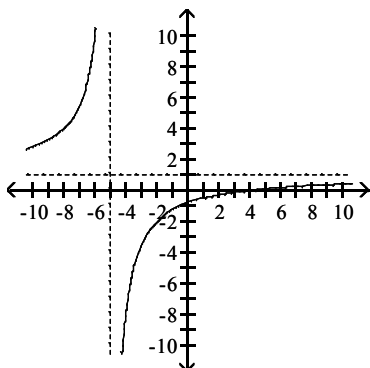


- 11) $(x + 2)(x^2 - 3x + 6) - 8$
- 12) $Q(x) = (2x^3 - 7x^2 + 6x - 15); R(x) = 45$
- 13) $f(x) = x^3 - 6x^2 - 1x + 30$
- 14) $f(x) = x^3 - 5x^2 + 4x - 20$
- 15) $-3i$
- 16) $1 + 2i, 1 - 2i$
- 17) $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6, \pm 9, \pm \frac{9}{2}, \pm 18$
- 18) 4, 6, -2
- 19) 1 or 3 positive; 0 negative
- 20) $(-\infty, -5) \cup (-5, 0) \cup (0, \infty)$
- 21) $x = 2, x = -4$
- 22) $y = -1$
- 23) None

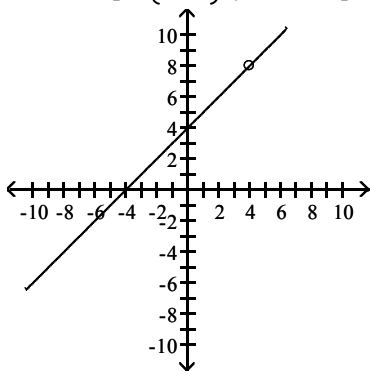
Answer Key

Testname: POLYNOMIAL AND RATIONAL FUNCTIONS TEST REVIEW

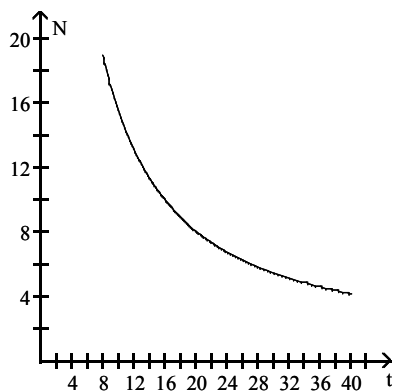
24) x-intercept: $(4, 0)$; y-intercept: $(0, -\frac{4}{5})$;



25) x-intercept: $(-4, 0)$, y-intercept: $(0, 4)$;



26)



$N(t) \rightarrow 0.1$ as $t \rightarrow \infty$.

27) $f(x) = \frac{2x^2 - 4x}{x^2 - 3x - 18}$

28) $(-\infty, -8) \cup (1, \infty)$

29) $[-2, 0] \cup [2, \infty)$

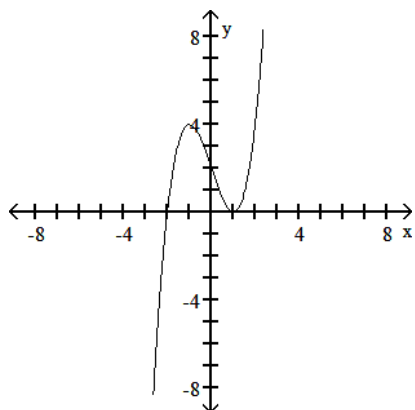
30) 4, 6; $(-\infty, 4) \cup (6, \infty)$

31) $(0.408, 2.449)$

Logarithm and Exponential Functions Review

Determine whether or not the function is one-to-one.

1)



2) $f(x) = x^2 + 7$

3) $f(x) = 5x^3 + 6$

If the function is one-to-one, find its inverse. If not, write "not one-to-one."

4) $f(x) = \frac{3}{x - 8}$

5) $f(x) = \sqrt[3]{x + 3}$

6) $f(x) = (x + 5)^2$

Solve the equation.

7) $(\sqrt{5})^x + 1 = 25^x$

8) $e^{4x} - 1 = (e^3)^{-x}$

Find the future value.

9) \$1972 invested for 12 years at 4% compounded quarterly

Solve the problem.

10) Find the required annual interest rate, to the nearest tenth of a percent, for \$1100 to grow to \$1400 if interest is compounded monthly for 7 years.

11) The decay of 938 mg of an isotope is given by $A(t) = 938e^{-0.022t}$, where t is time in years since the initial amount of 938 mg was present. Find the amount (to the nearest milligram) left after 96 years.

Solve the equation.

12) $\log_x 9 = -2$

$$13) \log(x - 5) 10 = 1$$

Use the properties of logarithms to rewrite the expression. Simplify the result if possible. Assume all variables represent positive real numbers.

$$14) \log_{16} \left(\frac{9\sqrt{m}}{n} \right)$$

Write the expression as a single logarithm with coefficient 1. Assume all variables represent positive real numbers.

$$15) (\log_a t - \log_a s) + 4 \log_a u$$

Use the change of base rule to find the logarithm to four decimal places.

$$16) \log_9 62.74$$

Solve the equation. Round to the nearest thousandth.

$$17) 4^{(x - 1)} = 22$$

$$18) 5e^{(4x - 1)} = 25$$

Solve the equation and express the solution in exact form.

$$19) \log(x + 10) = 1 + \log(4x - 3)$$

$$20) \log_9(x - 4) + \log_9(x - 4) = 1$$

Solve for the indicated variable.

$$21) f = i - k \ln t, \text{ for } t$$

Solve the problem.

22) How long must \$5300 be in a bank at 5% compounded annually to become \$9993.94? (Round to the nearest year.)

23) The growth in population of a city can be seen using the formula $p(t) = 10,124e^{0.004t}$, where t is the number of years. According to this formula, in how many years will the population reach 15,186? Round to the nearest tenth of a year.

24) Suppose $f(x) = 34.0 + 1.3 \log(x + 1)$ models salinity of ocean water to depths of 1000 meters at a certain latitude. x is the depth in meters and $f(x)$ is in grams of salt per kilogram of seawater. Approximate the depth (to the nearest tenth of a meter) where the salinity equals 37.

25) Suppose that $y = \frac{2 - \log(100 - x)}{0.29}$ can be used to calculate the number of years y for x percent of a population of 499 web-footed sparrows to die. Approximate the percentage (to the nearest whole per cent) of web-footed sparrows that died after 3 yr.

Answer Key

Testname: Logarithm and Exponential Functions Review

- 1) No
- 2) No
- 3) Yes
- 4) $f^{-1}(x) = \frac{8x + 3}{x}$
- 5) $f^{-1}(x) = x^3 - 3$
- 6) not a one-to-one
- 7) $\left\{\frac{1}{3}\right\}$
- 8) $\left\{\frac{1}{7}\right\}$
- 9) \$3179.31
- 10) 3.5%
- 11) 113
- 12) $\left\{\frac{1}{3}\right\}$
- 13) {15}
- 14) $\log_{16} 9 + \frac{1}{2} \log_{16} m - \log_{16} n$
- 15) $\log_a \left(\frac{tu^4}{s}\right)$
- 16) 1.8837
- 17) {3.230}
- 18) {0.652}
- 19) $\left\{\frac{40}{39}\right\}$
- 20) {7}
- 21) $t = e^{(i - f)/k}$
- 22) 13 yr
- 23) 101.4 yr
- 24) 202.1 m
- 25) 87%

Systems of Equations Review

Solve the following Systems using any method:

1) $8x + 36 = -4y$
 $-5x - 2y = 20$

2) $-3x + 4y = 24$
 $-6x = 22 - 8y$

3) $\frac{3}{2}x - \frac{1}{3}y = -18$
 $\frac{3}{4}x + \frac{2}{9}y = -9$

4) $x + 2y = 18$
 $4x + 8y = 72$

5) Jim wants to plan a meal with 82 grams of carbohydrates and 1260 calories. If green beans have 7 grams of carbohydrates and 30 calories per half cup serving and if french fried shrimp have 9 grams of carbohydrates and 190 calories per three-ounce serving, how many servings of green beans and shrimp should he use?

6) A student takes out two loans totaling \$11,000 to help pay for college expenses. One loan is at 7% simple interest, and the other is at 10% simple interest. The first-year interest is \$890. Find the amount of the loan at 10%.

7) In a chemistry class, 5 liters of a 4% silver iodide solution must be mixed with a 10% solution to get a 6% solution. How many liters of the 10% solution are needed?

Solve the system.

8) $2x + 4y + 10z = 105$
 $x + 2y + 5z = -21$
 $x + y + z = -5$

9) $x - y - 5z = -4$
 $y + 3z = 6$
 $x + y + z = 8$

Let u represent $\frac{1}{x}$, v represent $\frac{1}{y}$, and w represent $\frac{1}{z}$. Solve first for u , v , and w . Then solve the system of equations.

10) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -\frac{1}{5}$
 $\frac{1}{x} - \frac{1}{y} - \frac{9}{z} = \frac{37}{15}$
 $\frac{2}{x} + \frac{1}{y} + \frac{1}{z} = -\frac{2}{5}$

Find the product, if possible.

11) $\begin{bmatrix} -1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & -2 & 6 \\ 1 & -3 & 2 \end{bmatrix}$

Evaluate the determinant.

$$12) \begin{vmatrix} 2 & 0 & 0 \\ 7 & -9 & 0 \\ -9 & 5 & 7 \end{vmatrix}$$

Solve using Cramer's rule.

$$13) \begin{cases} 4x + 3y - z = 2 \\ x - 9y + 6z = 31 \\ 7x + y + z = 17 \end{cases}$$

Decompose into partial fractions.

$$14) \frac{9x - 31}{x^2 - 7x + 12}$$

$$15) \frac{26x^2 - 76x + 54}{(x - 5)(2x - 1)^2}$$

Graph

$$16) \begin{cases} y \leq 4x - 5, \\ y \geq -x \end{cases}$$

$$17) \begin{cases} 4y - x > -28, \\ y + 4x < 23, \\ 4x > y, \\ y < 0 \end{cases}$$

Answer Key

1) $(-2, -5)$

2) No solution

3) $(-12, 0)$

4) Infinitely many solutions

5) 4 half cups of beans and 6 three-ounce helpings of shrimp

6) \$4000

7) 2.5 L

8) No solution

9) $(2z + 2, -3z + 6, z)$

10) $(-5, 3, -3)$

11)

$$\begin{matrix} 3 & -7 & 0 \\ 4 & -14 & 14 \\ -126 & & \end{matrix}$$

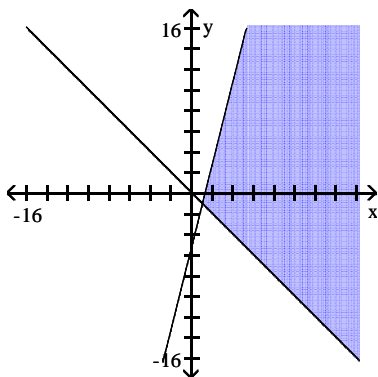
12) $\begin{bmatrix} 4 & -14 & 14 \\ -126 & & \end{bmatrix}$

13) $(1, 2, 8)$

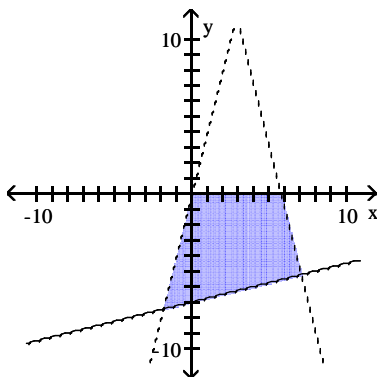
14) $\frac{5}{x-4} + \frac{4}{x-3}$

15) $-\frac{5}{(2x-1)^2} + \frac{5}{2x-1} + \frac{4}{x-5}$

16)



17)



Conic Sections Review

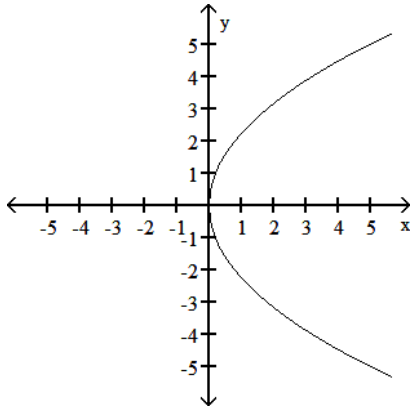
- 1) Graph: $y^2 - 5x = 0$
- 2) Find the focus and directrix: $y^2 = -28x$
- 3) Find the equation of a parabola given: Focus $(0, -6)$, directrix $y = 6$
- 4) Find the equation of a parabola given: Focus $(-3, -5)$, directrix $y = 9$
- 5) Find the vertex, the focus, and the directrix of the parabola: $(y + 2)^2 = 12(x + 4)$
- 6) Find the vertex, the focus, and the directrix of the parabola: $y^2 + 8x + 2y + 25 = 0$
- 7) A searchlight is shaped like a paraboloid of revolution. If the light source is located 2 feet from the base along the axis of symmetry and the opening is 14 feet across, how deep should the searchlight be?
- 8) Find the equation of a parabola with the following: Vertex: $(2, 3)$; horizontal axis of symmetry; containing the point $(18, 1)$
- 9) Find the center and radius for the circle: $x^2 + y^2 - 16x - 2y = -16$
- 10) Find the vertices and foci of the ellipse: $\frac{x^2}{25} + \frac{y^2}{4} = 1$
- 11) Find the vertices and foci of the ellipse: $36x^2 + 49y^2 = 1764$
- 12) Find the equation of an ellipse with the following: Vertices: $(-15, 0)$ and $(15, 0)$; length of minor axis: 14
- 13) Find the equation of an ellipse with the following: Foci: $(-1, 1)$ and $(-1, -5)$; length of major axis: 10
- 14) Find the equation of an ellipse with the following: Vertices: $(-2, -10)$ and $(-2, 2)$; endpoints of minor axis: $(-4, -4)$ and $(0, -4)$
- 15) Find the equation of the hyperbola: Center at $(0, 0)$; focus at $(0, 2\sqrt{5})$; vertex at $(0, 4)$
- 16) Find the equation of the hyperbola: Asymptotes $y = \frac{2}{9}x$, $y = -\frac{2}{9}x$; one vertex $(9, 0)$
- 17) Find the equation of the hyperbola: Vertices at $(0, 7)$ and $(0, -7)$; foci at $(0, 11)$ and $(0, -11)$
- 18) Find the vertices of the hyperbola: $36x^2 - 4y^2 = 144$
- 19) Graph: $4x^2 - y^2 = 4$

20) Graph: $9x^2 + 36y^2 = 324$

21) $\frac{(x - 2)^2}{9} + \frac{(y + 1)^2}{4} = 1$

Answer Key
CONIC SECTIONS REVIEW

1)



2) F: $(-7, 0)$; D: $x = 7$

3) $x^2 = -24y$

4) $(x + 3)^2 = -28(y - 2)$

5) V: $(-4, -2)$; F: $(-1, -2)$; D: $x = -7$

6) V: $(-3, -1)$; F: $(-5, -1)$; D: $x = -1$

7) 6.1 ft

8) $\frac{1}{4}(x - 2) = (y - 3)^2$

9) $(8, 1)$; $r = 7$

10) V: $(-5, 0)$, $(5, 0)$;
F: $(-\sqrt{21}, 0)$, $(\sqrt{21}, 0)$

11) V: $(-7, 0)$, $(7, 0)$;
F: $(-\sqrt{13}, 0)$, $(\sqrt{13}, 0)$

12) $\frac{x^2}{225} + \frac{y^2}{49} = 1$

13) $\frac{(y + 2)^2}{25} + \frac{(x + 1)^2}{16} = 1$

14) $\frac{(x + 2)^2}{4} + \frac{(y + 4)^2}{36} = 1$

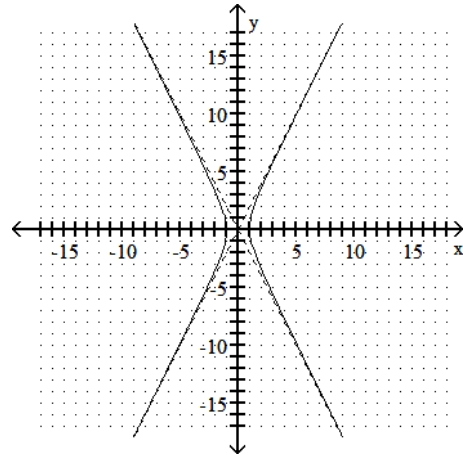
15) $\frac{y^2}{16} - \frac{x^2}{4} = 1$

16) $\frac{x^2}{81} - \frac{y^2}{4} = 1$

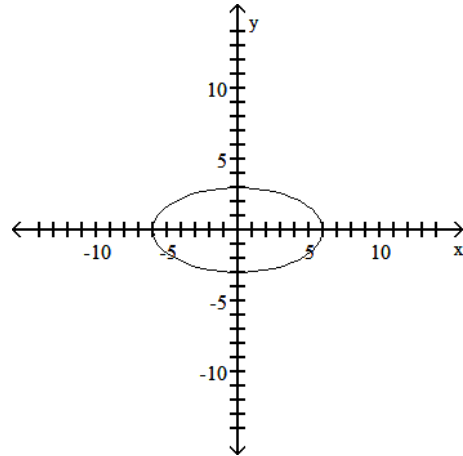
17) $\frac{y^2}{49} - \frac{x^2}{72} = 1$

18) $(-2, 0)$, $(2, 0)$

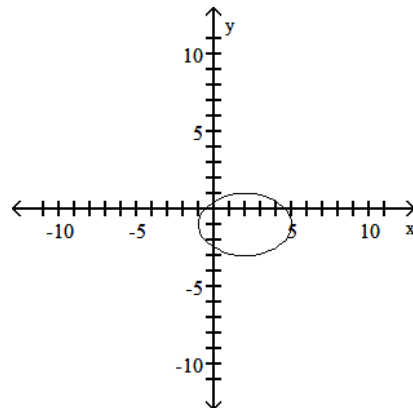
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20)



21)



Sequences and Series Review

- 1) Find the first 5 terms of the sequence: $a_n = 4n - 9$
- 2) Find the first 5 terms of the sequence: $a_n = \frac{5n - 1}{n^2 + 5n}$
- 3) Find the first 5 terms of the given sequence: $a_1 = 4, a_2 = 5, a_{n+1} = a_n + a_{n-1}$
- 4) Find the first 5 terms of the given sequence: $a_1 = -7, a_{n+1} = a_n + 3$
- 5) Find a_7 ; $a_n = (5n - 7)(6n - 5)$
- 6) Write an equation for the sequence: $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$
- 7) Evaluate: $\sum_{i=1}^4 (i^2 - 5)$
- 8) Evaluate: $\sum_{k=2}^5 \frac{1}{4k(k+1)}$
- 9) Find the 12th term of the arithmetic sequence: 6.1, 5.6, 5.1, ...
- 10) Find a_{37} , given $a_1 = 3$ and $d = -5$
- 11) Write a formula for S_n given: $a_1 = 21, d = -3$, and $n = 19$
- 12) The population of a species at the beginning of 1989 was 27,900. If the population decreased 250 per year, how many existed at the beginning of 2001?
- 13) Find the common ratio: 3, -9, 27, -81, 243, ...
- 14) Find the 10th term of the geometric sequence: 2, -6, 18, ...
- 15) Write the general formula for the geometric sequence: 49, 7, 1, ...
- 16) Find the sum of the first 10 terms in the geometric series $\frac{1}{45} + \frac{1}{15} + \frac{1}{5} \dots$
- 17) Find the sum, if it exists: $\sum_{i=1}^{\infty} 23 \left(\frac{1}{5}\right)^{i-1}$
- 18) Find the infinite sum if it exists: $0.42 + 0.0042 + 0.000042 + \dots$

Answer Key:

1) -5, -1, 3, 7

2) $\frac{2}{3}, \frac{9}{14}, \frac{7}{12}, \frac{19}{36}$

3) 4, 5, 9, 14

4) -7, -4, -1, 2

5) 3551

6) $\frac{n+1}{n+2}$

7) 10

8) $\frac{1}{12}$

9) 2.1

10) -177

11) -114

12) 24,150 people

13) -3

14) -486

15) $7^3 - n$

16) $\frac{1093}{45}$

17) $\frac{115}{4}$

18) $\frac{14}{33}$

Cumulative Review

1) Factor. $8m^{7/4} - 9m^{-1/2}$

2) Simplify: $(x^{-5}y^9)(x^{-2}y^7)^{-1}$

3) **Solve:** $\frac{x}{x-5} - \frac{5}{x+5} = \frac{50}{x^2-25}$

4) **Solve:** $\sqrt{2x+3} - \sqrt{x+1} = 1$

5) **Simplify:** $\frac{4 + \frac{2}{x}}{\frac{x}{4} + \frac{1}{8}}$

6) Simplify: i^{42}

7) Solve: $V = \frac{1}{m}\sqrt{2Vem}$, for m

8) **Solve:** $x^3 + 8 = 0$

9) Solve the equation: $2x^{-2} - 12x^{-1} + 10 = 0$

Solve and graph the inequality. Give answer in interval notation.

10) $-1 \leq \frac{x+1}{2} \leq 3$

Solve the rational inequality. Write the solution set in interval notation.

11) $\frac{x+13}{x+8} < 5$

Solve the inequality. Write the solution set in interval notation.

12) $(x+10)(x-8)(x+2) > 0$

13) $|5x-6| - 1 \geq 7$

14) Find the center-radius form of the equation of the circle having a diameter with endpoints $(-5, 1)$ and $(3, 7)$.

15) A biologist recorded 6 snakes on 20 acres in one area and 9 snakes on 45 acres in another area. Find a linear equation that models the number of snakes in x acres.

For the polynomial, one zero is given. Find all others.

16) $P(x) = x^3 - 3x^2 - 5x + 39$; -3

Find a polynomial of degree 3 with real coefficients that satisfies the given conditions.

17) Zeros of -3, -1, 4 and $P(2) = 15$

Give all possible rational zeros for the following polynomial.

18) $P(x) = -2x^4 + 5x^3 + 2x^2 + 18$

19) Graph the function.: $f(x) = \begin{cases} 3x - 2, & \text{if } x < 2 \\ 3x + 1 & \text{if } x \geq 2 \end{cases}$

20) Graph the function.: $g(x) = -\sqrt{x+2} + 1$

21) Compute and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$, for $f(x) = 8x^2 + 9x$

22) Divide. $\frac{-16x^3 + 8x^2 + 23x + 6}{-4x - 3}$

23) Graph: $f(x) = x^3 + 4x^2 - x - 4$

24) A \$128,000 trust is to be invested in bonds paying 9%, CDs paying 6%, and mortgages paying 10%. The bond and CD investment together must equal the mortgage investment. To earn a \$11,170 annual income from the investments, how much should the bank invest in bonds?

25) Find the number of years for \$6600 to grow to \$14,300 at 6% compounded quarterly. Round to the nearest tenth of a year.

If the function is one-to-one, find its inverse. If not, write "not one-to-one."

26) $f(x) = \frac{3}{x-8}$

27) Find the partial fractions for: $\frac{4x-20}{(x+5)(x-3)}$

Solve the equation and express the solution in exact form.

28) $\log_9(x-7) + \log_9(x-7) = 1$

Solve the equation.

29) $3(6-3x) = \frac{1}{27}$

30) $e^x - 3 = \left(\frac{1}{e^6}\right)^{x+2}$

Solve the equation. Round to the nearest thousandth.

$$31) 166(1.28)^{x/4} = 332$$

Solve the equation. Give the answer in exact form.

$$32) 5^{2x} + 3(5^x) = 28$$

Solve the equation.

$$33) \log(x + 7) 11 = 1$$

Solve the system.

$$34) 4x + 5y + z = -18$$

$$5x - 4y - z = 31$$

$$2x + y + 4z = -5$$

Give all solutions:

$$35) x^2 + y^2 = 41$$

$$x + y = -9$$

Answer Key

Testname: CUMULATIVE REVIEW

1) $m^{-1/2}(8m^{9/4} - 9)$

2) $\frac{y^2}{x^3}$

3) \emptyset

4) $\{3, -1\}$

5) $\frac{16}{x}$

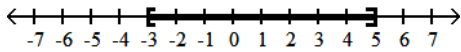
6) -1

7) $m = \frac{2e}{\sqrt{v}}$

8) $\{-2, 1 \pm i\sqrt{3}\}$

9) $\left\{1, \frac{1}{5}\right\}$

10) $[-3, 5]$



11) $(-\infty, -8) \cup \left[-\frac{27}{4}, \infty\right)$

12) $(-10, -2) \cup (8, \infty)$

13) $(\infty, -\frac{2}{5}] \cup \left[\frac{14}{5}, \infty\right)$

14) $(x + 1)^2 + (y - 4)^2 = 25$

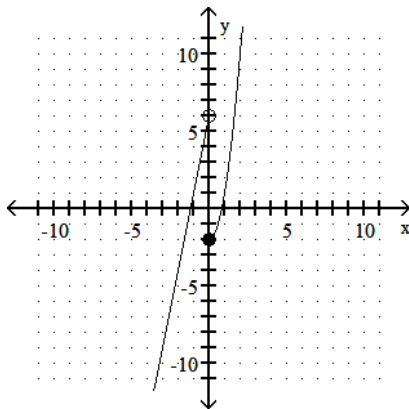
15) $y = \frac{3}{25}x + \frac{18}{5}$

16) $3 + 2i, 3 - 2i$

17) $P(x) = -\frac{x^3}{2} + \frac{13x}{2} + 6$

18) $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6, \pm 9, \pm \frac{9}{2}, \pm 18$

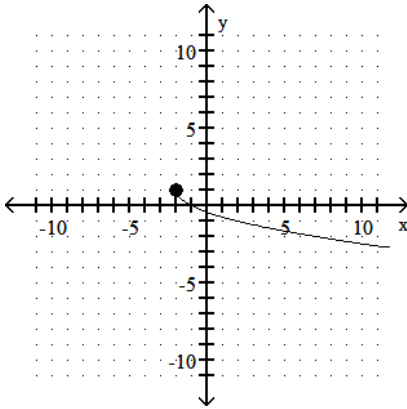
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Answer Key

Testname: CUMULATIVE REVIEW

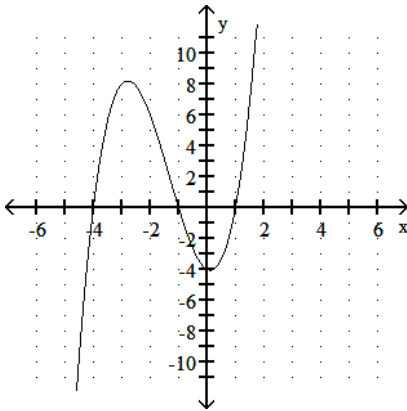
20)



21) $16x + 8h + 9$

22) $4x^2 - 5x - 2$

23)



24) \$31,000

25) 13.0 yr

26) $f^{-1}(x) = \frac{8x + 3}{x}$

27) $\frac{5}{x + 5} + \frac{-1}{x - 3}$

28) {10}

29) {3}

30) $\left\{-\frac{9}{7}\right\}$

31) {11.231}

32) $\{\log_5 4\}$

33) {4}

34) $\{(2, -5, -1)\}$

35) $\{(-4, -5), (-5, -4)\}$

Articles

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 - The University of Arizona's Biology Project website: <http://www.biology.arizona.edu/>
- ❖ Content was developed for Los Angeles Mission College College Algebra classes which use the Pearson textbook *College Algebra Graphs and Models* 6th Edition, by Bittinger/Beecher/Ellenbogen/Penna, and some of the content was derived from Pearson's TestGen testbanks for this textbook.
- ❖ The Open Educational Resource Textbook from OpenStax *Precalculus* by Abramson is referenced for students as a resource for content review.
- ❖ Brainology and Mindset article is by Carol Dweck
- ❖ The remaining articles are from the website: <https://phys.org/>