

Geometric Method for Calculating Fret Spacing By Eddie Ufford

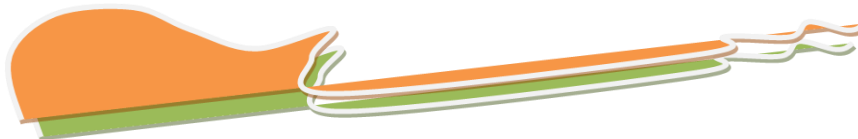
Description of Activity

- ☛ In this activity we are going to calculate the fret spacing for any given scale length using geometry and computer aided design and drafting software (CADD). This is part one of three separate MLAs dealing with custom fret board manufacturing. This MLA is the foundation for the other three. There are other ways to calculate fret spacing, but this is one of the few that is specifically geared geometry enthusiasts and CADD students.
- ☛ The purpose of this activity is to teach the students how to accurately calculate the fret board spacing using Computer Aided Design & Drafting software. This is one part of three MLA's that will walk one through the process to create their own custom fret board. In this first MLA, we are only locating fret lines. The physical size and shape of the fret board will be developed in the second MLA. In the third MLA, we will use the Laser Cutter/Engraver to test the placement of our fret lines and fret board shape onto a blank fret board. Finally in a fourth MLA, we will replace the standard $\frac{1}{8}$ " fret dots, with custom inlays.
- ☛ This activity is suitable for middle and high school students with experience with CADD software. While a printer is capable of printing out a template that then can be attached to a fret board blank for cutting; for my students we will be using our laser cutter for marking all the lines on our fret board material.

Learning Objectives:

(List measurable objectives)

- ☛ Students will calculate fret spacing using the "Tangent Line Method".
- ☛ Students will calculate fret positions and locations for various scale lengths.
- ☛ Students will use the "Rule of 18" for determining the distance needed to locate and mark the 1st Fret.
- ☛ Students will create a CAD file ie .DXF drawing that accurately places the locations for all desired frets to any given scale length within .00001 tolerance. Then proceed to use the laser cutter/engraver to laser down their lines onto a fret board blank.
- ☛ Student will use hand tools (calibers, protractors and bench rules) to measure an existing fret board for its size and shape.
- ☛ Students will develop a CAD drawing that will eventually be used for laser cutting new fret boards. The fret board design must include its size, shape, location of fret lines, fret dot markers and any desired inlay work.



Standards:

CCSS.Math.Content.HSA-CED.A.4
CCSS.Math.Content.HSG.GMD.B.4
CCSS.Math.Content.HSG.MG.A.3
CCSS.Math.Content.HSA.SSE.A.2
CCSS.Math.Content.6.NS.B.2
CCSS.Math.Practice.MP1
CCSS.Math.Practice.MP2
CCSS.Math.Practice.MP3
CCSS.Math.Practice.MP4
CCSS.Math.Practice.MP5
CCSS.Math.Practice.MP6
CCSS.Math.Practice.MP7
CCSS.Math.Practice.MP8
CCSS.Math.Content.HSG.GMD.B.4
High School: Modeling

Materials Required:

- ☛ A computer with CADD software, such as Rhino 3D, Fusion 360, AutoCAD, Solidworks, etc...
- ☛ Digital Calibers
- ☛ Printer, paper and spray on adhesive
- ☛ Blank fret board material
- ☛ Laser Cutter/Engraver (optional)

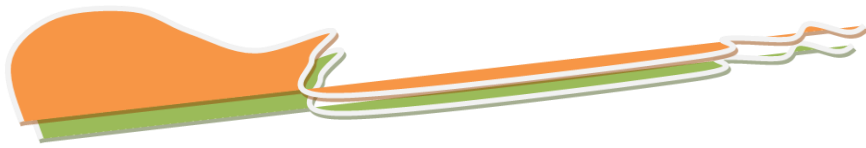
Safety:

- ☛ Review the list of safety concerns with traditional drafting tools.
- ☛ Review the safety procedure for laser cutter and engravers.

References:

- ☛ I first learned about this alternate geometric method “How to calculate the fret slot positions using the “tangent line method” from watching Jamie Boss’s YouTube video: <https://www.youtube.com/watch?v=lp-KclqVgQ> I credit him for showing me the basics to this geometric fret board calculation method. From there I used my CADD skills to develop this into lesson plans and MLA. Accordingly, my Woodworking and CAD students have been make their own fret boards for their bass guitars, electric guitars and tenor ukuleles since 2011 using Jamie Boss’s method.
- ☛ https://www.stewmac.com/How-To/Online_Resources/Learn_About_Guitar_and_Instrument_Fretting_and_Fretw/Scale_Length_Explained.html





Activity:

Before beginning this CAD activity, it is required that one watches the short video <https://www.youtube.com/watch?v=lp-KclqVgQ> from Jamie Boss. How to calculate the fret slot positions using the tangent line method. The author of the video calls it a Tangent Line Method, yet with the power of CAD software and infinite zooming capabilities the referred to line it is very close to being a “Tangent” line, but technically not. As a **tangent line** is a straight **line** that touches the arc or circle at only one point, but it extended would not cross.

When we zoom in, the “tangent” line is passing through two points that are extremely close together, 0.001470”. **Spelled out 0.00147” is One hundred forty-seven, one hundred thousandths of an inch.** For the sake of an argument, is that amount worth quarrelling over? Jamie Boss didn’t have the power of CAD for his video and to give credit to the originator, I will also call this method of fret spacing the “Tangent Line Method”. You decide, is there a better name for this procedure other than tangent line method?

For this activity, we are going to start by drawing a very large and exact right triangle via our CAD software. I am using the CAD software package called Rhinoceros 3D. It is a professional grade high powered 3D Non-uniform rational Basis spline (NURBS) modeler that also does a fantastic job at flat two dimensional drawings. You can adapt this method to use with any CAD software of your choosing. One should set their drawing precision to a minimum of 5 or more decimal places. Also, one needs to use their drawing aids such as Osnaps or constraints to locate exact end points, center points, intersections and perpendicular lines.

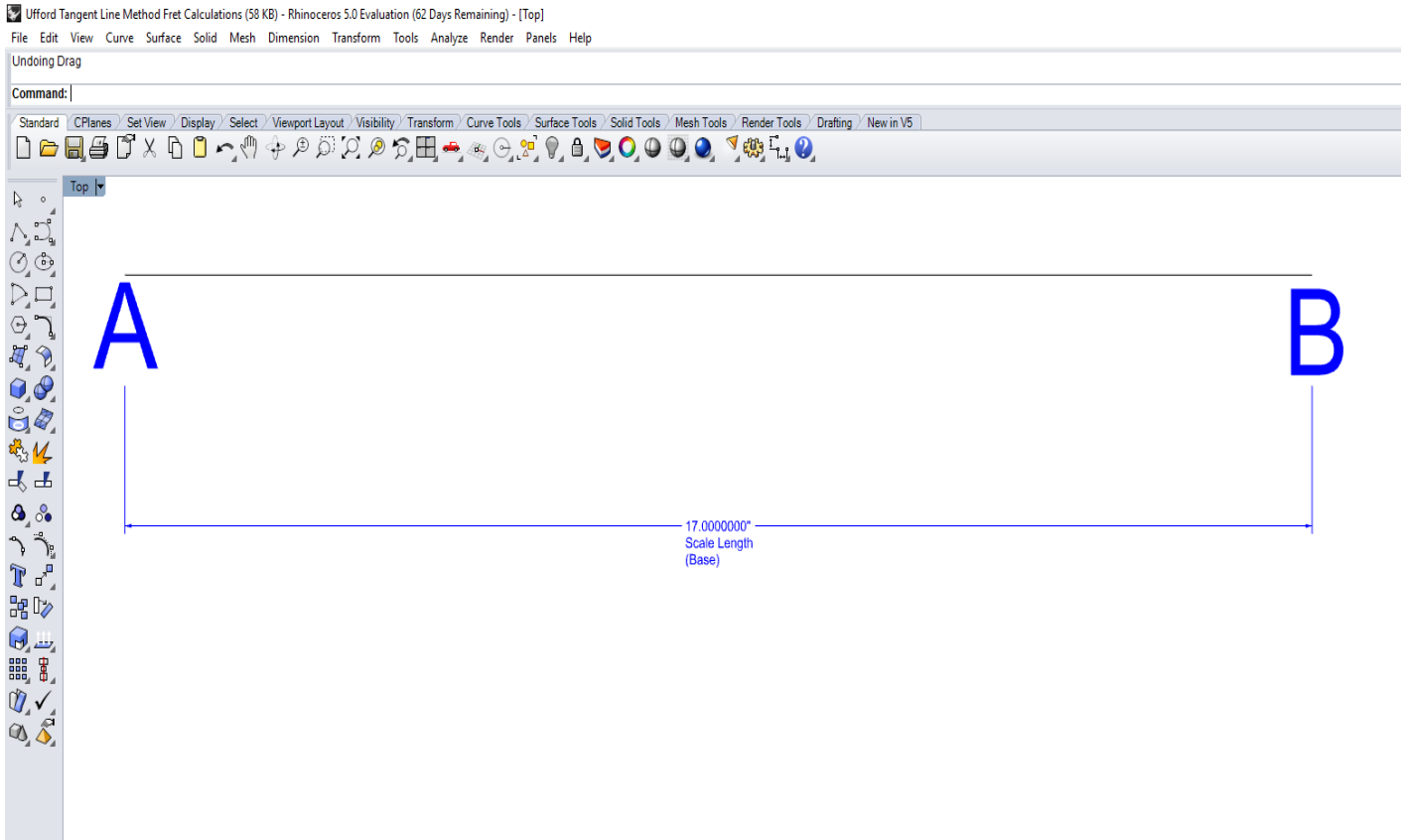
We are going to start by drawing a right triangle. The base of the triangle is the given or desired scale length. The height of the triangle needs to be calculated by dividing the scale length by the Rule of 18, (*scale length/17.817= triangle height*). For this lesson, the height of the triangle may also be referred to as the Nut Line or Fret Zero. Finally, the hypogenous will be drawn by connecting the two end points.

For this lesson we are going to use the scale length of 17.000”, standard tenor ukulele. But this method will work for any scale length. Also, an accuracy checker will be placed at the first octave, which is also the 12th Fret. It is located exactly one half of the scale length (8.500” for our example). Additionally, we will be laying out 18 fret lines for our fret board, but more could be done if desired.



Step 1 Draw the triangle's Base to match the Scale Length

- ☛ For this tutorial we are going to calculate the fret wire spacing for a tenor ukulele scale length of 17.000". But you may use this method for any given scale length. Simply replace the tenor ukulele scale length with your desired scale length and run your calculations based from that.
- ☛ If you are unfamiliar with scale lengths, read this article from StewMac, understanding scale length. https://www.stewmac.com/How-To/Online_Resources/Learn_About_Guitar_and_Instrument_Fretting_and_Fretw/Scale_Length_Explained.html
- ☛ We will start by drawing the base line AB for our triangle. AB is equal to the desired scale length (17.000"). From point A of line AB, this will be where the Nut Line Height begins. At point B of line AB is where the Hypotenuse "Tangent Main Line" will eventually intersect. *For this tutorial we will designate the left as fret zero, point A, and the right end for the saddle as point B.*



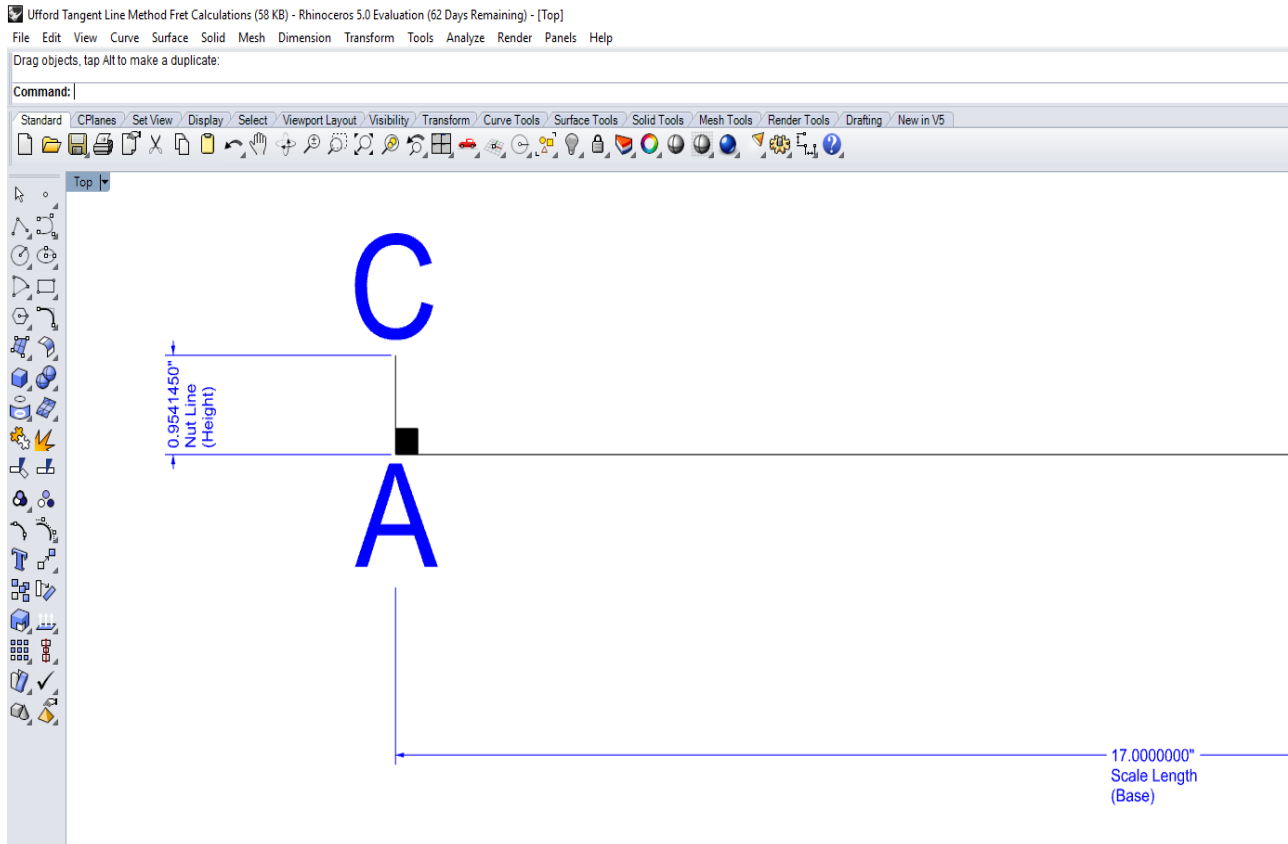


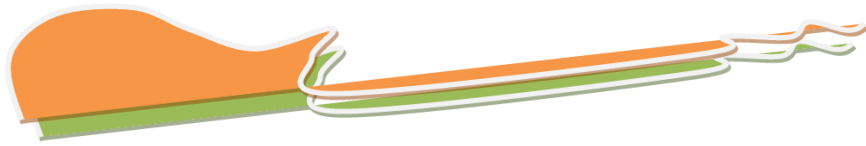
Step 2 Set the height of the right triangle

Calculate and draw a line AC. It is equal to the triangle's height. Before we can find the height of the triangle we must first calculate the distant needed to locate the first fret. Consequently, this is also the same amount for the nut line height. See example below.

Therefore, using the refined "Rule of 18" (17.817), divide the scale length by this number. For example, our ukulele uses a 17 inch scale length. Thus, $17'' / 17.817 = 0.95414491777''$. This is the height of our triangle. Accordingly, it is also the same distance from the "nut line" or fret zero to the 1st Fret.

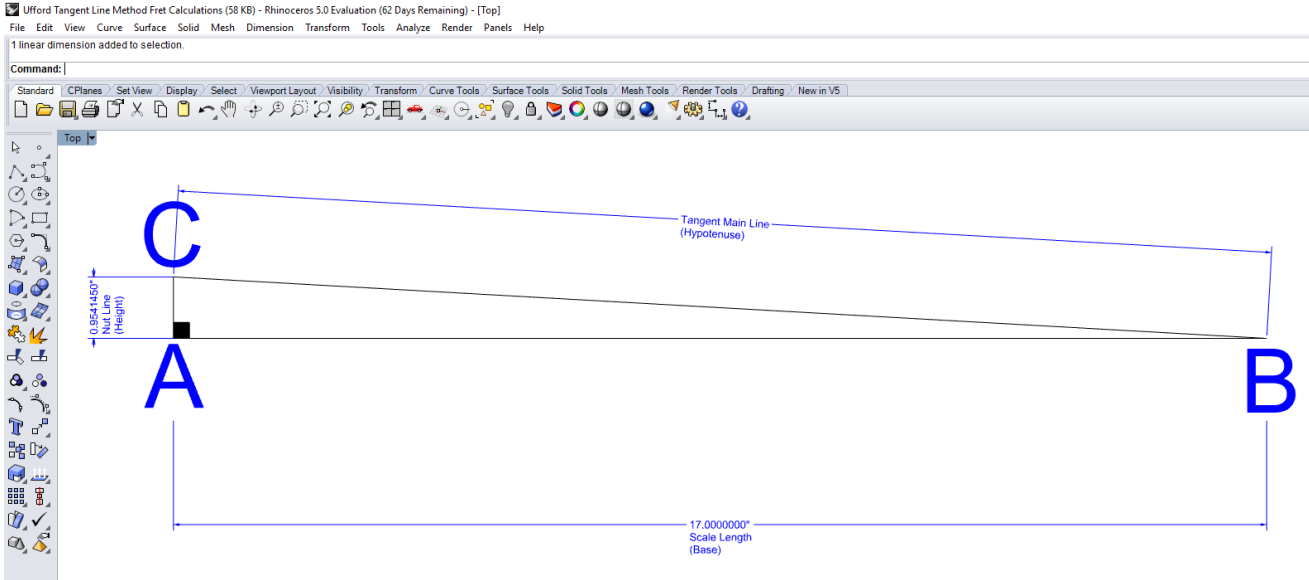
Therefore, from point A on base line AB, draw upwards a perpendicular line. The end of this line will be labeled point C.





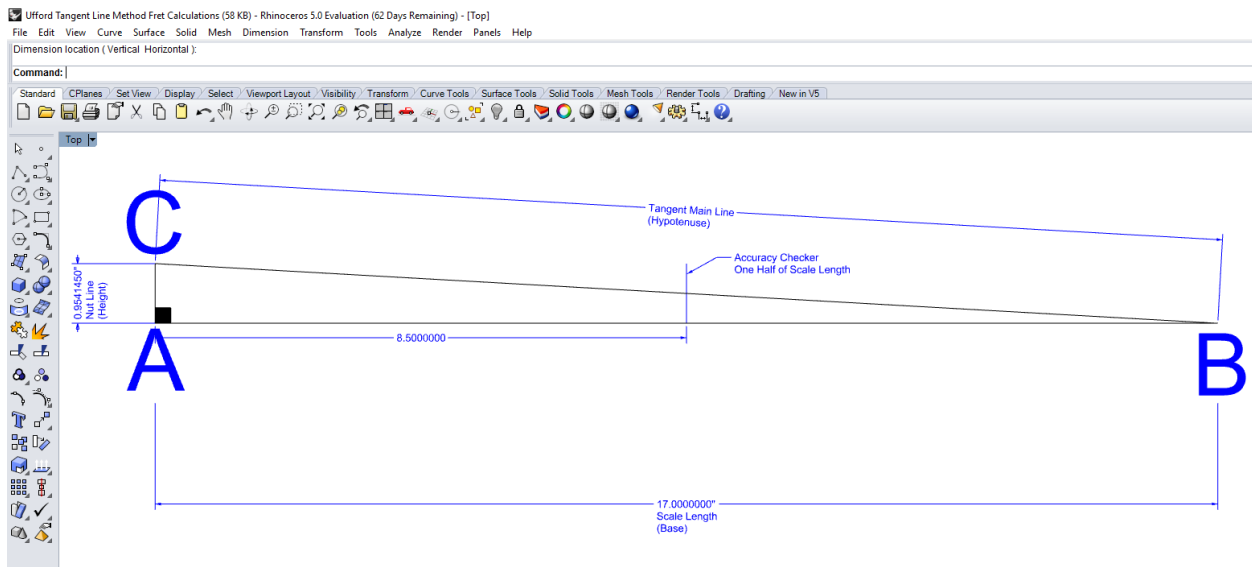
Step 3 Set the Tangent Line

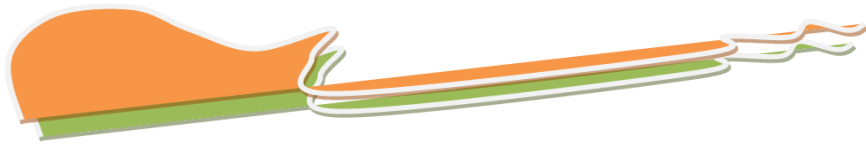
Draw the hypotenuse “Tangent Main Line” by connecting points C and B with a straight line. You'll end up with a long triangle as shown here. Congratulations, you have just setup the triangle foundation for a tenor ukulele fret board spacing.



Step 4 Add the Accuracy Checker

Before we go any further, we are going to place an accuracy checker onto our base line. Bisect base line AB and then draw a short Perpendicular line upwards. The 12th fret must land exactly at this intersection on the base line AB; this also marks the first octave.

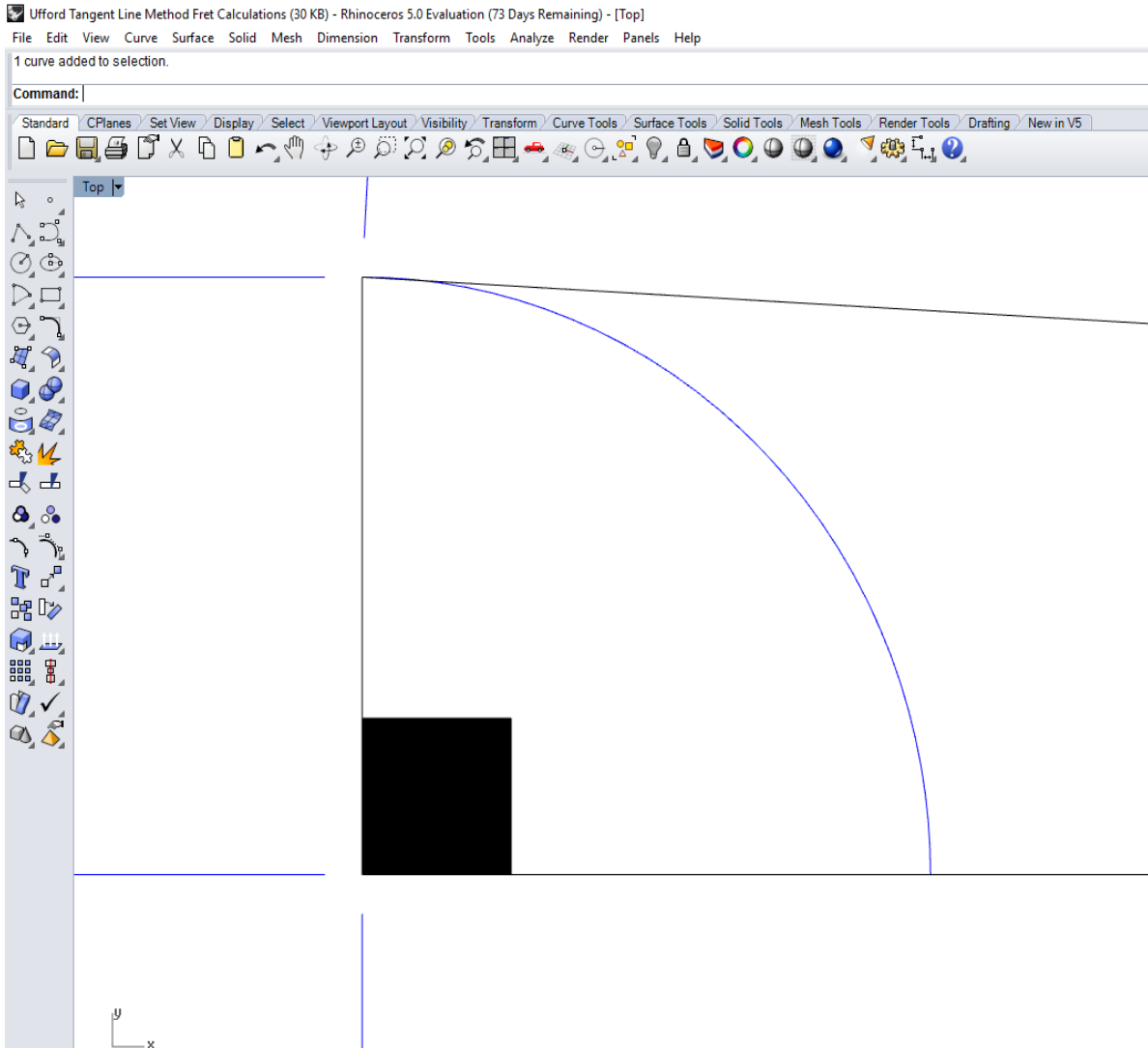


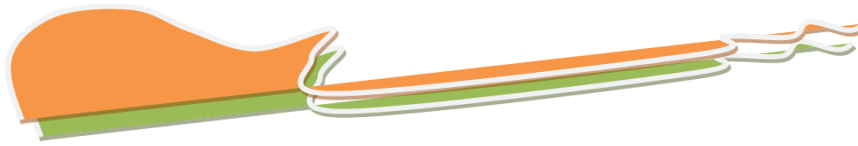


Step 5

Swing the first Arc

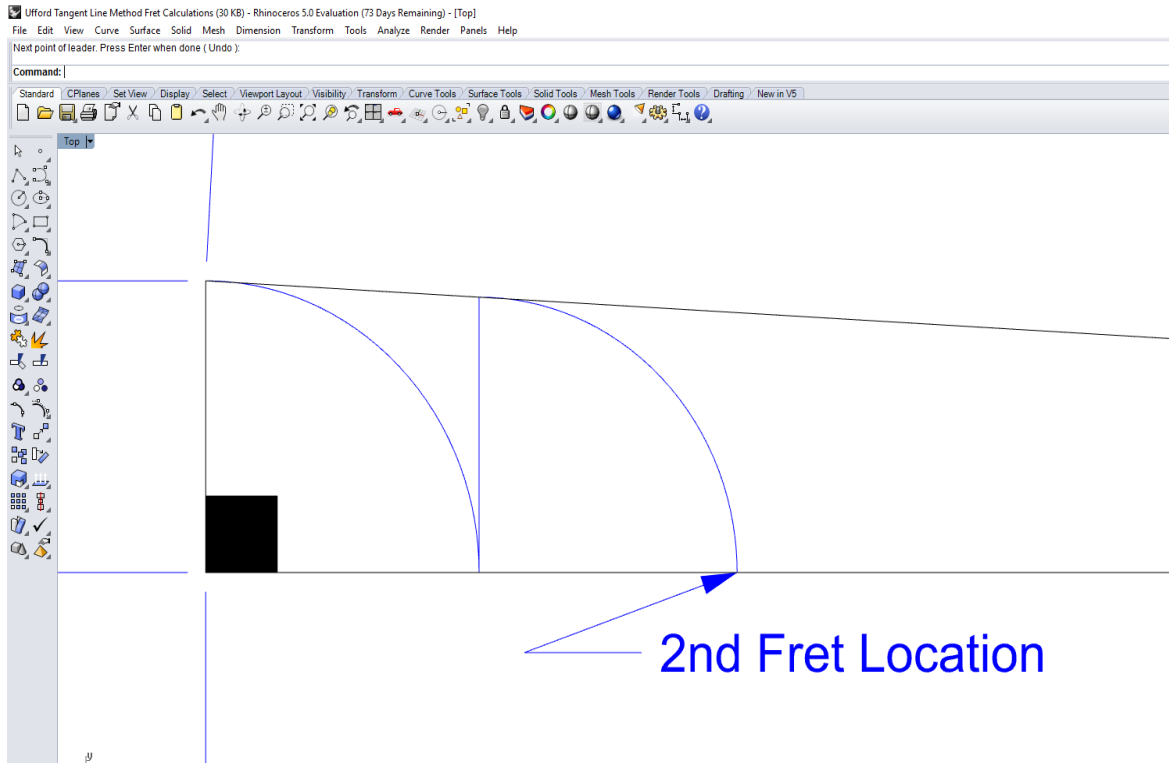
Locate Fret position #1. Using the Arc command, start an arc starting at point A, that spans line AC. Swing/Draw the Arc so it intersects line AB. This intersection point on Line AB is the exact distance and location for Fret #1.

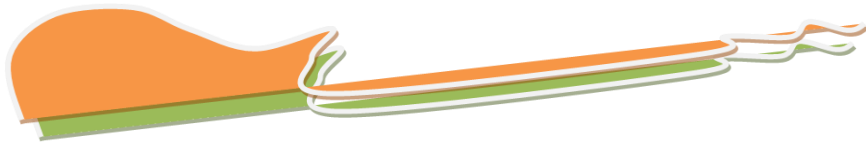




Step 6 Intersect and Repeat

Locate fret position #2. Draw a perpendicular line upwards to intersect the hypotenuse line from Fret #1 location. Now Swing/Draw a new Arc. This time starting at Fret #1 that spans the new perpendicular line height. Swing the Arc so it intersects Line AB. You have just located the 2nd Fret position.



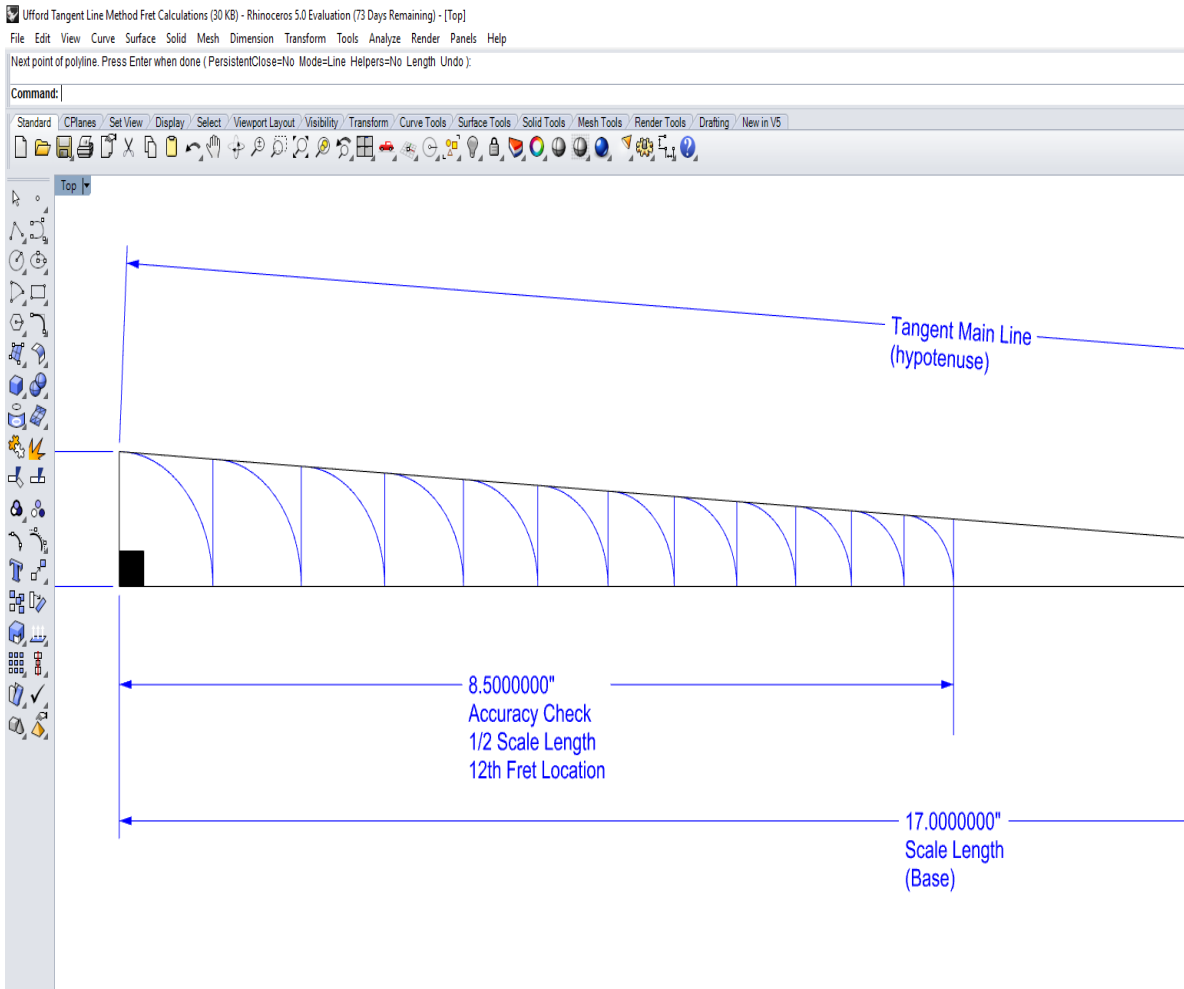


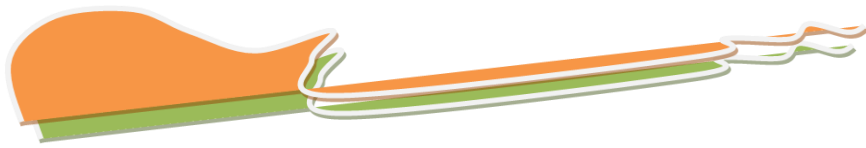
Step 7



More Intersect and Repeat

Repeat the actions in Step #6 to locate the position of frets #3 thru #12.



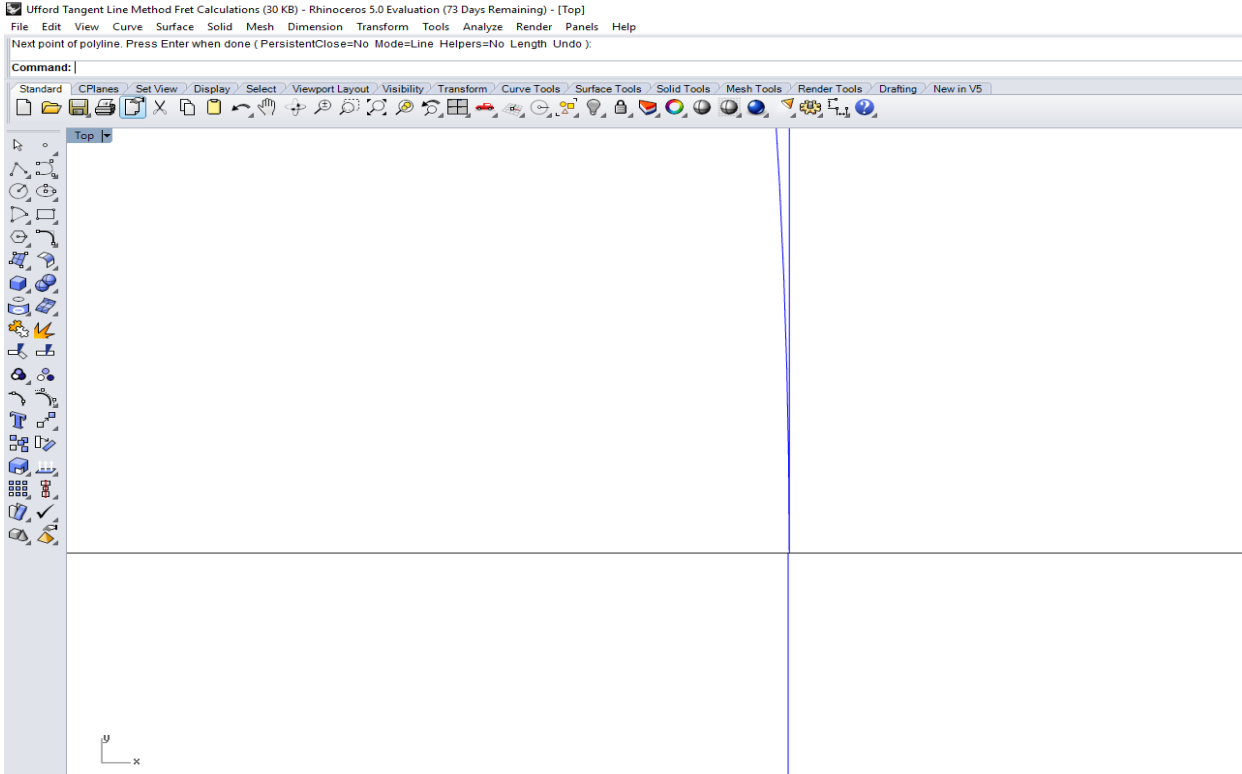


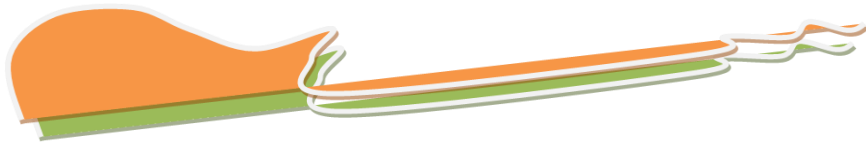
Step 8



Check the accuracy checker

Zoom into the 12th Fret location for an Accuracy Check (100x zoom).



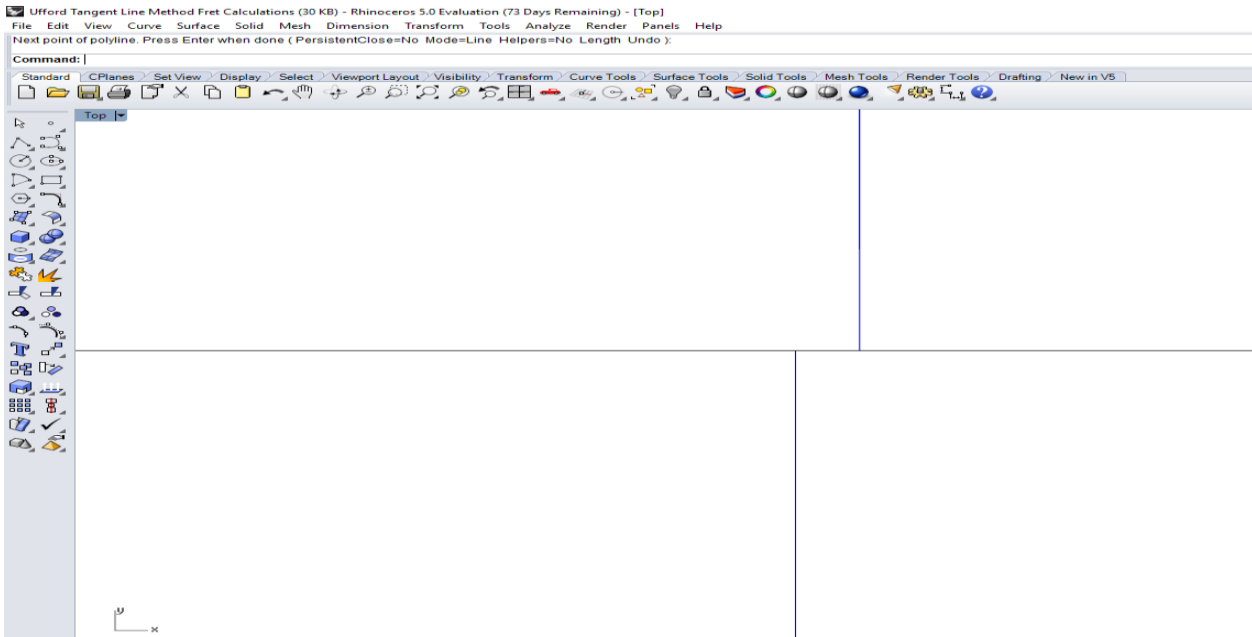


Step 8b

Zoom In

Zoomed into 10,000x magnification

How far are we off?

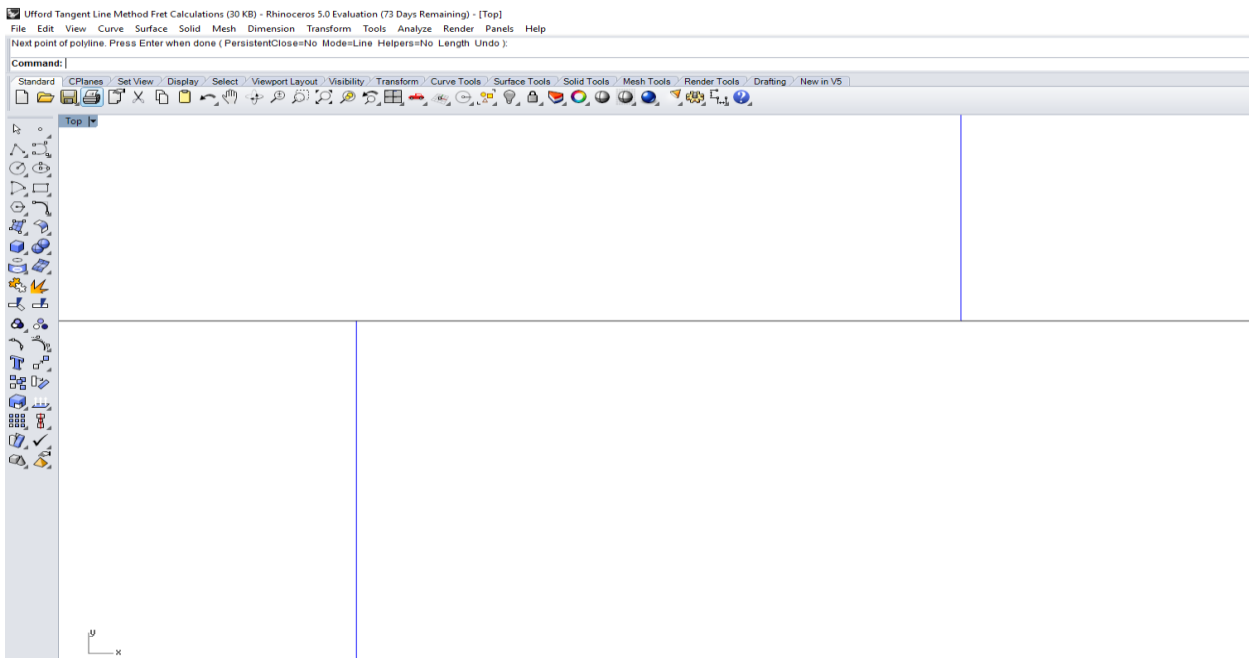


Step 8c

Zoomed in even further

Zoom 100,000x magnification.

Use your Dimensioning tools to check the accuracy.

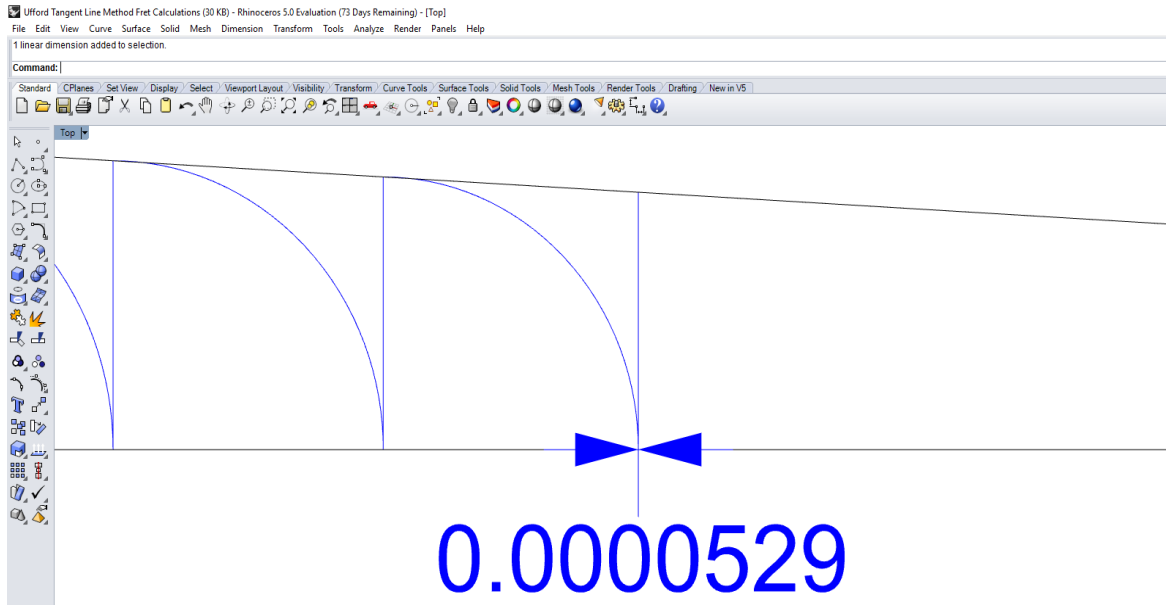




Step 9 How far are we off?

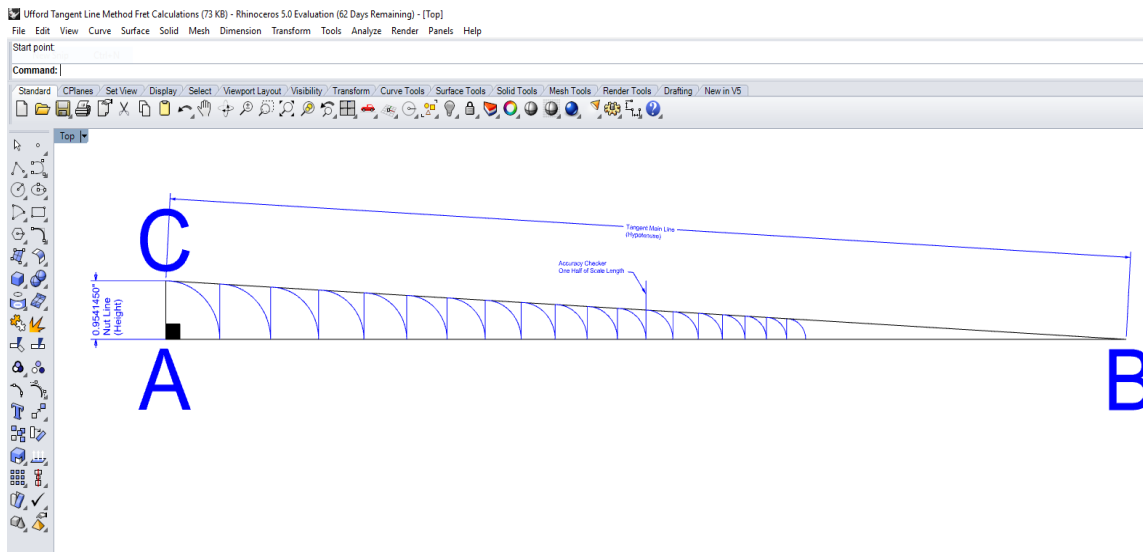
Use the Dimension command to check the accuracy of where the 12th fret should be in relationship of where the Tangent line method located it.

For this example, we discovered that we are off by 0.0000529". **Spelled out, that is "five hundred twenty-nine, ten-millionths of an inch"**. Question is: Is that an acceptable tolerance for locating fret positions?



Step 10 Start Swinging Arcs Again

Locate the remaining frets. One could continue from there (being off by five hundred twenty-nine ten-millionths of an inch). A better solution is to simply move over to the true 12th fret location and restart the Tangent Line Method.

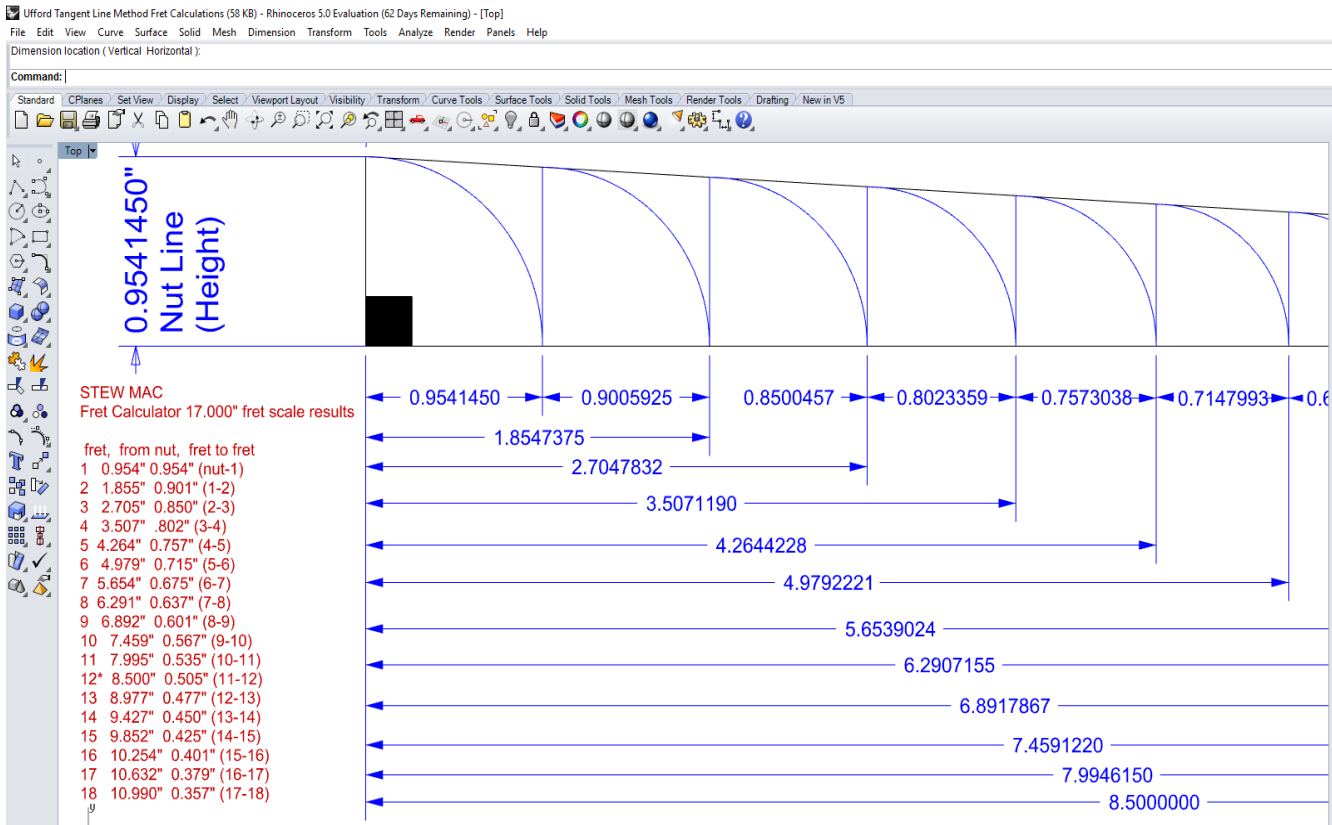




Step 11

Dimension and Compare your results

Dimension and Compare all fret locations with online Stew Mac's Fret Calculator.



Step 12

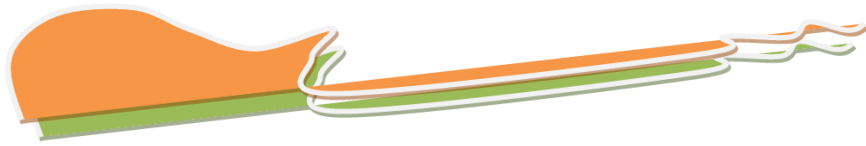
Save and Export to .DXF

Save and make a backup copy of your CAD file. One will also need a .DXF file for importing into another drawing.

In MLA activity #2 we shall be transforming the fret locating into an actual tenor ukulele fret board. We will then program our laser cutter and engraver to mark and cut out our sample fret board.

Main Learning Points:

- ☛ This method can be used to do calculate fret positions for any scale length.
- ☛ The vibrating string length or scale length is equal to the base of the triangle.
- ☛ The height of the triangle is derived by dividing the scale length by the rule of 18.
- ☛ The Rule of 18 refined value is 17.817
- ☛ The hypotenuse is made by connecting points B and C with a straight line
- ☛ The mid-point of the base line is the 12th fret location; which is also the 1st octave.
- ☛ A tangent line only touches a single point, and if extended will not cross. A secant line crosses at two points. Technically, this should be termed the Secant line method. But now we are literally splitting the thickness of a human hair.



Quiz:

Questions:

Circle the best answer(s).

1. Who is the creator of the Tangent Line Method YouTube Video?
 - a) Jamie Boss
 - b) Tom Singer
 - c) Eddie Ufford
 - d) Tom Doniphon
2. What is the refined number for the Rule of 18?
 - a) 18.017
 - b) 18.000
 - c) 17.817
 - d) 17.810
3. From the lesson above, what measurement is used for the base of the triangle?
 - a) The distance from the nut to the 1st fret.
 - b) The scale length of a tenor ukulele
 - c) The distance from points C to B.
 - d) 17.000"
 - e) Both B and D.
4. From the lesson above, how does one calculated the height of the triangle?
 - a) Rule of 18/the give scale length=height of triangle
 - b) $18.817/17$ =height of triangle
 - c) Scale length/2= height of triangle
 - d) Answers A & B
 - e) None of the Above
5. From the lesson above, how does one draw the hypotenuse?
 - a) By finding "X".
 - b) Connecting points B & C with a straight line.
 - c) By bisecting line AB
 - d) By bisecting line AC
6. From the lesson above, where is the accuracy checker place?
 - a) At the first octave
 - b) At the mid-point of line AB.
 - c) At 8.5000"
 - d) All the above
7. What is the distance from the nut to the first fret?
 - a) Given Scale Length/Rule of 18





- b) $17'' / 17.817 = 0.95414491777$
c) Both A & B
d) None of the above
8. What is the significance of the 12th fret?
a) It is the exact mid-point for the vibrating string length (VSL). Some people will call it the scale length.
b) It defines the first octave
c) Both A & B
d) None of the above.
9. Which answers best describes tangent?
a) An entity such as a straight line or arc that touches a curve at one point, but if extended does not cross it at that point.
b) An entity such as a straight line or arc that intersect an arc or circle at two points.
c) An entity such as a line that intersects four points of a curve.
d) None of the above.
10. What is of 0.0000529''?
a) Five Hundred Twenty-Nine, ten millionths of an inch.
b) Five Hundred Twenty-Nine, millionths of an inch.
c) Five Hundred Twenty-Nine, one hundred thousandths of an inch.
d) Five Hundred Twenty-Nine, ten thousandths of an inch.

Answer Key:

1. Who is the creator of the Tangent Line Method YouTube Video?
e) **Jamie Boss**
f) Tom Singer
g) Eddie Ufford
h) Tom Doniphon
2. What is the refined number for the Rule of 18?
i) 18.017
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m) The distance from the nut to the 1st fret.
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o) The distance from points C to B.
p) 17.000''
q) **Both B and D.**
4. From the lesson above, how does one calculate the height of the triangle?
r) **The give scale length/Rule of 18 = height of triangle**
s) $18.817/17 = \text{height of triangle}$
t) Scale $\text{length}/2 = \text{height of triangle}$





- u) Answers A & B
v) None of the Above
5. From the lesson above, how does one draw the hypotenuse?
w) By finding "X".
x) **Connecting points B & C with a straight line.**
y) By bisecting line AB
z) By bisecting line AC
6. From the lesson above, where is the accuracy checker place?
aa) At the first octave
bb) At the mid-point of line AB.
cc) At 8.5000"
dd) **All the above**
7. What is the distance from the nut to the first fret?
ee) Given Scale Length/Rule of 18
ff) $17'' / 17.817 = 0.95414491777$
gg) **Both A & B**
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8. What is the significance of the 12th fret?
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nn) An entity such as a straight line or arc that intersect an arc or circle at two points.
oo) An entity such as a line that intersects four points of a curve.
pp) None of the above.
10. What is of 0.0000529".
qq) **Five Hundred Twenty-Nine, ten millionths of an inch.**
rr) Five Hundred Twenty-Nine, millionths of an inch.
ss) Five Hundred Twenty-Nine, one hundred thousandths of an inch.
tt) Five Hundred Twenty-Nine, ten thousandths of an inch.
uu) Point zero, zero, zero, zero, five, two and nine.

Reviewing Faculty Cohort Members:

- ☛ Allen Androkites
- ☛ Matt Peitzman
- ☛ Alex Moll